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Statistical Properties of Art Pricing Models

Course: Executive Master in Art Market Studies, University of Zurich

Master Thesis

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State of Authorship:

I hereby certify that this Master Thesis has been composed by myself and describes my own work, unless otherwise acknowledged in the text. All references and verbatim extracts have been quoted, and all sources of information have been specifically acknowledged. This Master Thesis has not been accepted in any previous application for a degree.

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1 Introduction

In November 2012 the painting *No. 1 (Royal Red and Blue)* of the American painter Mark Rothko was auctioned at Sotheby's in New York (fig.1). It is a wonderful painting that had an exciting pre-sale estimate range of USD 35 to 50 million, which is not at all explainable with any materiality argument. The price seems to be detached from the tangible ingredients, as long as immaterial ingredients like the artist's reputation or the importance of the period in the œuvre of the artist are not valued. The painting has been marketed with its uniqueness in the artist's development and career, with its perfect provenance and "aura".[1] The latter is the most difficult component to explain and document, and its perception may change over time, since it is related to the *zeitgeist*. The auction of Mark Rothko's painting was a success; the work was sold for USD 67 million, one third over the upper estimate. The painting was recorded in Skate's Top 5000 database at USD 75'122'496 including the buyer's premium and stands at position 13 of the 5'000 most expensive works recorded at auction.[2] The difference between the hammer price of USD 67 million and the price including the buyer's premium corresponds to the fee Sotheby's earned. Additionally there is also a seller's premium for the auction house of a few percentages, which is however negotiable for expensive works such as *No. 1 (Royal Red and Blue)*. Without having to do great math it is obvious that the marketing campaign paid-off.

Gertrude Stein said, "A work of art has either a value or a price".[3] In her terminology the value of the work is related to the perception by knowledgeable persons in art history, i.e. art critics, curators, academic researchers in art history and knowledgeable art collectors. The price, on the other hand, is the exchange value of a good between two parties. It is real, and most often related to a money transfer. The two concepts of value and price may indeed be different, but it is doubtful whether they are completely detached: A highly valued work of art is costly, but whether a work traded at a high price has a corresponding art historical valuation is more questionable. Most of the living artists have to undergo the *litmus* test of their actual valuation.

In this thesis we will concentrate on the price and the pricing of artworks by means of statistical methods. Often assumptions have to be made concerning the rules of price developments. These assumptions will be tested on a limited set of

[1] See the video from Sotheby's website, www.sothebys.com/en/auctions/2012/contemporary-art-evening-auction-n08900/videos.html, last access on 20.1.2013.
[2] www.skatepress.com, last access on 20.1.2013.
[3] Gertrude Stein (1874–1946) was an American writer, poet and art collector; citation from Alicia Legg (Ed.), *Sol LeWitt*, Museum of Modern Art, New York, 1978, p.164.

publicly available data. In the following, artworks will be regarded as part of a homogeneous portfolio. Although, we know that apart from prints and photography artworks are unique and heterogeneous.

Figure 1 Mark Rothko, *No. 1 (Royal Red and Blue)*, 1954, oil on canvas; 288.9 x 171.5 cm, from: http://qzprod.files.wordpress.com/2012/11/mark-rothko_s-1954-no-1-royal-red-and-blue-1.jpg?w=1024&h=1760, last access on 15.4.2013.

The thesis starts with an introduction in Chapter 2 to the methods of the art price index construction. In Chapter 3 we discuss different commercially available price indices. We compare two price estimates for a single artwork by Ed Ruscha to the pre-sale estimates and the realised hammer price at auction. Neither of the indices has made a convincing proposal for the realised price paid by the buyer.
For a third index we confirm that the time series is smoothed by construction. By a de-smoothing methodology originally applied to real estate and hedge fund returns we show that the underlying volatility is substantially higher than the reported standard deviation.

The main take-away from these results is the reminder to be careful when using an art price index without knowing the underlying methodology. Knowledge of the methodology allows to judge on the outcome and to decide how far it should be included in any expectation or decision.

In Chapter 4 the distributions of different pricing models are investigated: Skate Art Market Research's *Art Asset Pricing Model* (AAPM), the pricing of Campbell and Wiehenkamp (2009) in the determination of the expected loss for art backed loans and the pricing of artworks by McAndrew and Thompson (2007) as collateral value. We transform Skate's art asset pricing model into the form of a hedonic regression and find that the expected price movement over time is the consumer price inflation. The model also contains an irrational premium which reflects the buyer's willingness to over- or underpay the fair value of the artwork. This premium may or may not occur, and is therefore not predictable. In the hedonic regression we assign it to the idiosyncratic error term.

In the other two pricing models we put the focus on the price distribution. We apply the distribution used in Campbell and Wiehenkamp (2009) to the repeat-sales data from Skate's Art Market Research and find that although the fitted function is well shaped and passes three statistical tests to a high degree, the data is biased due to the index construction and contains too few negative returns.

Skate's Art Market Research is a consultancy company among many. However, the company is exceptional compared to other commercial art service firms in the sense that it makes data on sold artworks available on its website for free and downloadable in Excel-format.[4] Throughout the academic literature the most valuable good is the data set each researcher has gathered for his or her studies. The individual results can hardly be checked or confirmed by others with the same data since they are treated confidentially and proprietarily. Thus it may happen that,

[4] cf. note 2.

depending on the data, different findings occur with similar methods for the same question under investigation. As an example we name the *Underperformance of Masterpieces.* Mei and Moses (2002) found that the future excess price of masterpieces, i.e. expensive artworks, is lower than for the other artworks, whereas Renneboog and Spaenjers (2013) found a higher performance for higher priced artworks compared to others.[5] We will come back to this specific topic in Chapter 4.2.

Finally, the methodology of McAndrew and Thompson (2007) considers the pre-sale estimate of the auction house (or from any other evaluator) as *true* fair value and attempts to estimate the price distribution based on the ratio of the hammer price to the pre-sale auction house estimate, and to include the bought-ins. Bought-ins are lots that are not sold at auctions due to a reserve price set by the seller, and the work is “bought in-house” by the auctioneer. The highest bid remains unpublished, the price estimate and the tag “Not sold” are the only publicly available information on bought-ins. This incomplete information does not correspond to their importance for auction houses and collectors:

1. In approximately 20% of the cases a sale takes place immediately after the auction with the price again negotiated between the seller, the auction house and the buyer. Often this sale is not published.
2. An unsold artwork at auction is banned from the art market for a quite a while or can only be offered at a lower price.[6] It is referred to as *burned*.
3. The reputation of the auction houses depends on the sale rate, i.e. the percentage of works sold compared to the total number of lots offered: The higher the sale rate the higher the confidence of potential consignors that the auction house is able to sell the works at the estimated prices.

It is therefore worthwhile to consider bought-ins in the distribution of auction prices. We apply the model of McAndrew and Thompson (2007) to the auction results for drawings, paintings and photographs of Ed Ruscha and find that the model fits well to this data sample. We determine a distribution for the bought-ins which we did not find in this explicit form in the literature.

Chapter 5 contains the conclusion and lists possible extensions related to the topics discussed in the main text.

[5] For a review of earlier studies see Ashenfelter and Graddy (2003).
[6] For a study on the future value of unsold works at auction see Beggs and Graddy (2008).

2 Index construction

The methods used for the construction of the price development of an artwork rely on principles for the estimate of market prices of heterogeneous goods. Real estate valuation is maybe the most prominent example and a forerunner for the application of the index construction to artworks. Often, researchers that work in the field of real estate valuation also apply the same methodology to art pricing.[7] The price construction of heterogeneous goods usually involves the characteristics of the individual items and the development over time.

The characteristics contain measurable and descriptive attributes of the artwork, such as artist, technique, subject, date or period of creation and dimensions. It is usually assumed that the characteristics do not change and stay the same over time. The provenance can also be an important factor. Especially for potentially looted art and for flight art during the Second World War, the complete owner's history is crucial information. Provenance is less important for contemporary art, although the glory of a prominent owner is used in the marketing when a work or a collection comes to auction.[8]

The time development is related to changes in the economic environment, to the availability of collectors and their capability and willingness to invest in art. Anecdotally, sales at auction are driven by the three D's: Death, divorce and debts, i.e. need for money. The first two incidents can reasonably be assumed to be independent from economic environmental changes, whereas outstanding debts often go with missing liquidity, e.g. in a financial crisis. Mostly a liquidity crunch does hit a broader group of wealthy people who may then be forced to sell artworks or at least have a restricted capability to buy more artworks. As a consequence the prices drop without any change in the qualitative aspects of the works. With excess money available, the mechanism goes in the other direction and prices rise. Independent from the economic situation, a repositioning of the artist in the art historical context also influences the price development.

The two main methods commonly used for index construction are hedonic regression (HR) and repeat-sales regression (RSR).

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[7] See e.g. William N. Goetzmann, http://www.brg-expert.com/media/bio/213_Goetzmann_William_CV.pdf, last access on 3.3.2013.

[8] See the magazine for the auction of the Yves Saint Laurent and Pierre Bergé art collection at Christie's on 23–25 February 2009, http://www.christies.com/images/email/jan09_ysl_en/ysl_feb09.pdf, last access on 3.3.2013.

2.1 Hedonic regression

Hedonic regression *controls* the characteristics of the objects, and the residual is then said to be the *characteristic-free* contribution to form a price index.[9] This regression can be as simple as the distinction of the artist, size of the artwork and number of editions (for photography), or can be run over almost 100 variables.[10] Although the latter likely gives a higher degree of numerical precision than the former, the interpretation is not straightforward and becomes less clear when more variables are considered. The method needs price records and as precise information on the artwork as possible. Usually this means that only auction data can be considered, since transactions involving galleries which are mostly active on the primary market and art dealers for the secondary market are rarely available publicly. It is estimated that auctions cover roughly 50% of all the transactions in the art market.[11] Therefore hedonic regressions serve as a proxy for the art market but do not represent the entire market.

Mathematically, hedonic regression for an observed price P_{it} of an artwork i at time t can be expressed as

$$p_{it} = \ln(P_{it}) = p_i + p_t + \varepsilon_{it} \,, \tag{2.1}$$

where $\ln(P_{it})$ is the natural logarithm of P_{it}, p_i is the contribution from the unique character of the artwork, assumed to be fixed over time, and therefore without time index t. p_t stands for the contribution to the aggregate index of price movements valid for all different artworks considered, and ε_{it} is the work- and time-specific error term which is assumed to have zero mean, i.e. it vanishes on average.[12]

The term p_i is represented as

$$p_i = \beta \cdot x_i + \zeta_i \,,$$

where x_i contains the information on the characteristics of the artwork i, β is the universal parameter to be fitted to the data and ζ_i is again an error term. We obtain

$$p_{it} = \beta \cdot x_i + p_t + \widetilde{\varepsilon}_{it} \,,$$

[9] Ginsburgh et al. (2006), p. 953.
[10] Example for the first variant: For Ed Ruscha with 545 auction results from 1987 to 2012, the descriptive statistic with the three variables: Number of editions for photographs, size and an art market index can explain up to 65% of the auction prices. See Appendix A.
Example for the second variant: For 32 Western artists with 3'291 observations from 1992 to 2007, the descriptive statistic with 96 variables can explain 80% of the prices. See Gawrisch (2008).
[11] McAndrew (2012), p. 55.
[12] See Appendix B for a remark on the choice of the natural logarithm as price index function.

with $\tilde{\varepsilon}_{it} = \zeta_i + \varepsilon_{it}$. For later use we have a closer look at p_t, which results from the time series of all sales of the involved artworks and can be estimated from the average of the realised prices at time t after the deduction of the characteristics,[13]

$$\hat{p}_t = \frac{1}{n_t} \cdot \sum_{i=1}^{n_t} \left(p_{it} - \beta \cdot x_i \right), \tag{2.2}$$

where $\hat{p}_t$ is an estimator for p_t, and n_t is the number of sold works at time t.[14] It represents a price component that is free of the characteristics, unspecific for any artwork that takes part in the regression. Nevertheless, this price component contains a selection bias due to the fact that only sold works are taken into account. The bought-ins are left aside. In a study on auction data of French Impressionists McAndrew and Thompson (2007) found a bought-in rate of 30%. In Chapter 4.3 we will see how they attempted to overcome this shortcoming in their pricing model. The illustrative representation in eq.(2.2) assumes that the ordinary least square (OLS) estimator can be used. For cases where the OLS estimator is inappropriate more tedious approaches are needed, see e.g. Ginsburgh et al. (2006) and Goetzmann et al. (2010b).

Also noteworthy is the effect of adding a new sale p_{js} to the regression. Most likely, the regression will lead to other coefficients $\tilde{\beta}$ for the characteristics due to the heterogeneity of the artworks. From eq.(2.1) it is obvious that $\hat{p}_t$ will change although the artwork j is sold at time $s \neq t$. Due to the change $\beta \rightarrow \tilde{\beta}$ the common price development changes in *every* time period. This effect is called revision bias and may also occur in the case of a re-classification of a work, e.g. from Contemporary Art to Post-Modernism Art, or if new information concerning the chain of owner is included.

Revision bias matters in case art price indices are published.[15] Due to the inclusion of fresh auction results the price history will be changed. In Chapter 3.2 we will have a look how artnet, an art consultant company and auction data provider, deals with this problem.[16]

A way to overcome the shortcoming of the revision bias, at least partly, is the method of repeat-sales regression.

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[13] Ginsburgh et al. (2006).
[14] For practical use, the time t stands usually for a time period of one year instead for a single point in time. The one-year period will be assumed in the analysis unless stated differently.
[15] The corresponding issue for real estate is studied in Deng and Quigley (2008).
[16] www.artnet.com, last access on 1.3.2013.

2.2 Repeat-sales regression

In this regression only artworks are considered that have been sold at least twice. We look for the return between the two sales. Be the work i sold at time t as well as again later on at time T. From eq.(2.1) we have

$$p_{iT} - p_{it} = p_T - p_t + \varepsilon_{iT} - \varepsilon_{it} \text{ ,}$$

where the characteristics related term p_i vanishes since we assume that the character of the artwork does not change over time. We have therefore a characteristic free set of equations, apart from the residuals, over all holding periods of distinct artworks. The equation is then written as[17]

$$p_{iT} - p_{it} = r_i = \sum_{k=t+1}^{T} \left(\mu_k + \varepsilon_{ik} \right),$$

with r_i the return from the holding period $T-t$ of artwork i and μ_k the universal price movement in period k during the holding period. The determination of the estimator $\hat{\mu}$ goes along the same lines as for $\hat{p}_t$ in the hedonic regression in eq.(2.1).

The selection bias increases with repeat-sales regression since the fraction of included trades is reduced significantly compared to the method of hedonic regression. Goetzmann et al. (2010b) found 1349 resales out of 6661 works sold at least once. Skate's Top 5000 database contains 869 resales. The former yields a resale ratio of 20% out of all works considered, where we do not know the exact number of transactions. The latter has a resale ratio of 17% out of all trades.
The revision bias is still inherent in the approach, but does not affect the whole time series as it does in hedonic regression. In case a new resale is included, the price movement μ changes for the holding period of the newly added trade.

The holding period shows considerable differences depending on the data sample used. In Mei and Moses (2002) the holding period is indicated as "on average 28 years".[18] Their data sample starts in 1875 and lasted until 2000 by the time of publication. The Skate's Annual Art Investment Report for 2012 shows as an average holding period a substantially lower holding period: 8.4 and 9.3 years for resales in 2011 and 2012 respectively.[19] The report covers the resales from the believed 5'000 most expensive artworks and starts in 1989 with the first resale. A calculation over all the resales gives an average holding period of 8.5 years. Therefore, the addition of an actual resale is expected to change return history in the repeat-sales index of the last 10 years.

[17] Mei and Moses (2002).
[18] Ibid., p. 1657.
[19] Skate's Art Market Research, Annual Art Investment Report 2012, Part 1, p. 7.

3 Price indices

In this Chapter we have a look at three commercially available indices. We discuss the methodologies and compare two price estimates for a single artwork of Ed Ruscha to the pre-sale estimates and the realised hammer price at auction. For a third index we confirm that the series is smoothed and apply a de-smoothing methodology to determine the underlying volatility.

3.1 Artprice

In this section we show that Artprice's indicator for a single artwork by Ed Ruscha fails to predict the auction price realised in March 2013 even though it shows a large increase since November 2000.

Artprice is a publicly listed French company under the major control of Thierry Ehrmann.[20] It had a market capitalisation of around USD 260 million as of 31 December 2012 and is the second largest publicly traded art service company (behind Sotheby's).[21] It provides art services such as research, analyses and access to their proprietary auction price database. In case enough data for an artist is available, Artprice applies its Artprice Indicator®.[22] The goal of the indicator is to give a present value for a work, which has been auctioned. The indicative price comes with a rating system for the quality of the estimate.

The exact price mechanism is not published. The methodology papers tell that different index types are used: The highest precision includes all characteristics in a hedonic regression and needs five auction results per year which match the required criteria. In case of poor data the procedure is then reduced step by step. The lowest level still relies on a single artist's works; no aggregation of different artists is applied.

The characteristics used are signature, size, medium, period the works date from, material and technique used and the place of sales. In case information on the characteristics is missing, the regression is restricted to the medium. If there is still not sufficient information available, all media are included from the artist except the prints, and in a last step, prints are also considered. The procedure may be changed from year to year depending on the data available, which is then reflected in the rating system. It is explicitly mentioned that some features are not included, namely: State of preservation, origin, subject and framing.

Depending on the applied procedure, the indices are named differently: Artwork Price Index, Medium Price Index or Artist Price Index. This differentiation is

[20] www.artprice.com, last access on 24.2.2013
[21] Skate's Art Market Research, Annual Art Investment Report 2012, Part 2, p. 27.
[22] Artprice Indicator Methodology, web.artprice.com/indicator/howto?l=en, last access on 20.2.2013.

considered in the rating system called degree of relevance and indicated by a system of up to five stars. Results with less than three stars are not made available.[23]

The price indicator like other hedonic regression models suffers from selection bias since only sold works are considered. However, in the artist and medium price index development, the ratio of bought-ins to the total amount of works is shown; and therefore can be taken qualitatively into consideration.[24]

As an example we take a closer look at the price indicator of the painting *Anchor Stuck in Sand* (1990) from the artist Ed Ruscha (fig.2).[25] The artwork was sold on 15 November 2000 for USD 60'000 with an estimate between USD 65'000 and USD 85'000 at Sotheby's in New York. The work was offered again at auction on 5 October 2012 with an estimate between USD 700'000 and USD 900'000 at Phillips de Pury & Company in New York. The starting bid was USD 500'000 and the painting was not sold.[26] *Anchor Stuck in Sand* was on offer again only five months later on 7 March 2013 for an estimate between USD 600'000 and USD 800'000, again at Phillips in New York.

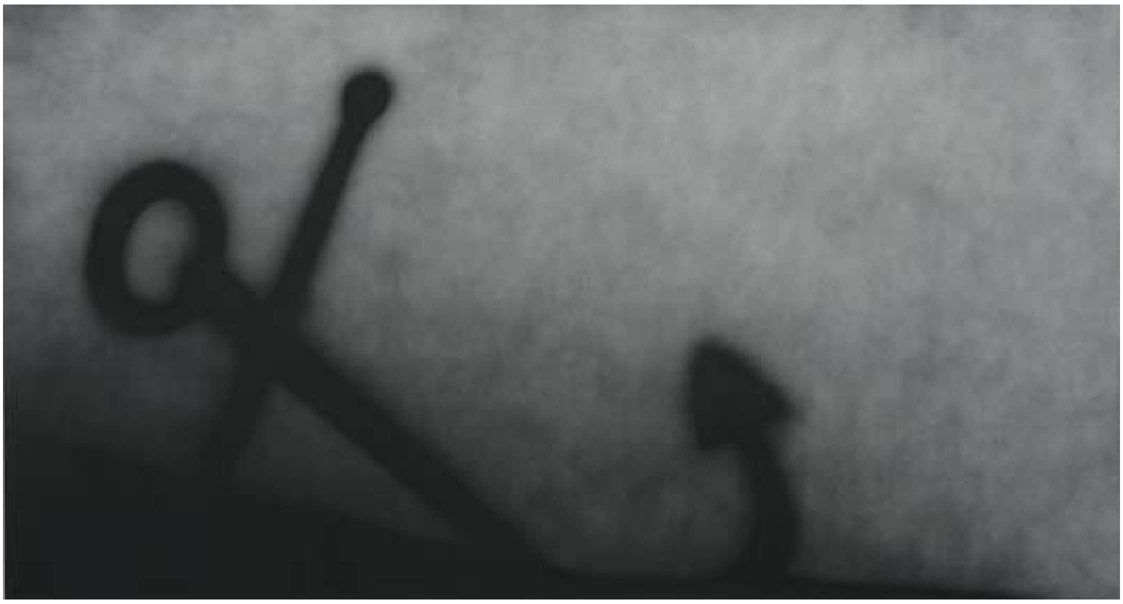

Figure 2 Ed Ruscha, *Anchor Stuck in Sand*, 1990, acrylic on canvas; 153 x 283.1 cm, from: http://www.liveauctioneers.com/item/11426722_ed-ruscha-anchor-stuck-in-sand-1990, last access on 1.3.2013.

[23] See Appendix C for a description of the method.
[24] See Figure 17 and the discussion in Chapter 5 for a possible quantitative approach.
[25] An introduction to Ed Ruscha (American, born 1937) follows in Chapter 4.3.1.
[26] http://www.liveauctioneers.com/item/11426722_ed-ruscha-anchor-stuck-in-sand-1990, last access on 1.3.2013.

This time it was still not sold at auction, but immediately afterwards it was sold privately for USD 550’000 including the buyer’s premium.[27] Artprice calculated the hammer price of USD 456’500.

The Artprice Indicator® for *Anchor Stuck in Sand* was calibrated at the end of the year 2000 to the hammer price of USD 60’000. It increased until the end of 2012, when the subsequent auction took place, to USD 197’287 (see fig.3). Interestingly, the price indicator development had the highest rating until the end of 2011, and then it broke down significantly. The indication was far below the estimates made at auction. Even if we doubt the assumed performance in 2012 due to the lower estimate quality, the gap between the price indication and auction estimates is large.

Artprice Indicator

Year	€ $	Growth rate	Relevancy
1993	25,065.93	-	★★★★★★
1994	15,052.73	-39.95 %	★★★★★★
1995	31,870.81	+111.74 %	★★★★★★
1996	17,491.21	-45.12 %	★★★★★★
1997	24,641.2	+40.88 %	★★★★★★
1998	24,826.48	0.75 %	★★★★★★
1999	43,999.88	+77.23 %	★★★★★★
2000	60,000	+36.37 %	Hammer Price
2001	87,531.42	+45.89 %	★★★★★★
2002	161,942.66	+85.01 %	★★★★★★
2003	141,399.45	-12.69 %	★★★★★★
2004	153,496.24	+8.56 %	★★★★★★
2005	182,406.59	+18.83 %	★★★★★★
2006	251,447.24	+37.85 %	★★★★★★
2007	424,726.21	+68.91 %	★★★★★★
2008	310,866.04	-26.81 %	★★★★★★
2009	291,541.29	-6.22 %	★★★★★★
2010	315,450.21	+8.2 %	★★★★★★
2011	242,304.75	-23.19 %	★★★★★★
2012	197,286.93	-18.58 %	★★☆☆☆☆

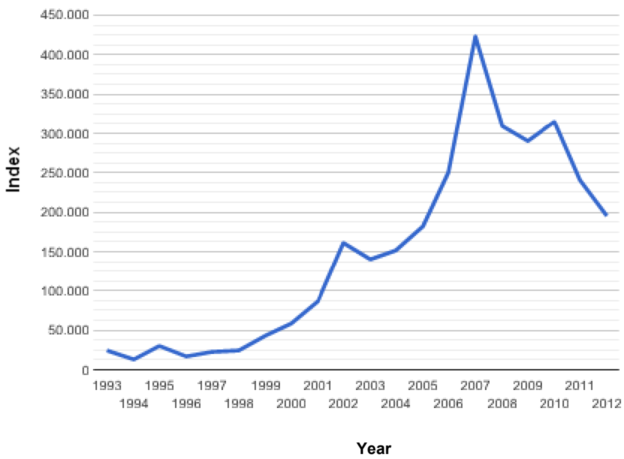

Figure 3 Artprice Indicator® for Ed Ruscha’s *Anchor Stuck in Sand* (1990). The indicator is calibrated in 2000 to the hammer price of USD 60’000. The price estimate is calculated back to 1993 and the forecast ends in 2012 at almost USD 200’000 based on auction results not specified otherwise. The stars correspond to a rating concerning the quality of the indicator.[28] Table and Chart are from: www.artprice.com, last access on 1.3.2013.

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[27] See http://www.phillips.com/Xigen/file.ashx?path=\\diskstation\website\Media\Auction\auctionResultsFile_NY010213.pdf, last access on 28.3.2013.

[28] Contrary to the statements made in the methodology paper, the maximum rating has six instead of five stars, and the index is published even with two stars only. Information from Artprice (Email from 4.3.2013): “Unfortunately we are in the middle of upgrading our Artworks Indexator, and the "star" system is right now having a little bug. We'll fix it as soon as possible”.

3.2 artnet

In this section we find that artnet's grouping of comparable works to Ed Ruscha's *Anchor Stuck in Sand* (1990) is questionable, and that there are no price changes after a sharp increase in 2001.

artnet is a publicly listed German company under the major control of Hans Neuendorf and operationally led by his son, Jacob Pabst.[29] It had a market capitalisation of approximately USD 19 million as of 31 December 2012, much less than Artprice's market capitalisation.[30] It is an online platform for gallery owners and collectors buying, selling and researching artworks. For clients artnet provides access to their proprietary auction price database and calculates indices according to the artnet Index Methodology for approximately 75% of the sold lots recorded in the database.[31] The individual indices are finally aggregated to broader market indices, such as the artnet C50™ Index.

The price indices are calculated by means of a mixed or hybrid model of a hedonic regression and a repeat-sales regression, i.e. the repeat-sales regression is a "nested case" of the hedonic regression.[32] The core of the model is to extend the repeat sale regression to sets of *comparables* in order to get a broader database. Comparables consist of groups of works by a single artist and are defined by experts based on "appraisal principles and art historical knowledge".[33] I.e. the specific knowledge lies in the identification of the comparables and less in the methodology, which is documented in a detailed manner. The corresponding logarithmic price of an artwork i belonging to the comparable set s and sold at time t may be written similar to eq.(2.1):

$$p_{ist} - \overline{p}_{ist} = p_s + p_t + \varepsilon_{ist} \,,$$

where the price p_{ist} is corrected by the average $\overline{p}_{ist}$ of the prices from the works belonging to the comparable set s sold at time t, and p_s represents the characteristics of the comparable set s instead of the contribution from the single artwork i. The underlying assumption is the equivalence of the works in the comparable set, and that the artworks within a set are priced on a similar level.

The prices are estimated on a monthly basis. In case no sale has taken place in the current month, a zero return is assumed. If new data is added, the regression for

[29] cf. note 16.
[30] cf. note 21.
[31] artnet Indices White Paper (Version 18 July 2012), http://www.artnet.com/analytics/reports/artnet-indices-white-paper, last access on 1.3.2013.
[32] Ibid., p. 16.
[33] Ibid., p. 3.

the price history changes as discussed in Chapter 2.1. In order to prevent that the past values of the indices are adjusted too often and only for small amounts, the parameters are calculated with a confidence level. When new data is considered, the estimated parameters of an index are compared with and without the new sale data. If the adapted parameters lie outside the boundaries given by the confidence interval, the index is updated. In case the estimated values are still within the previous range of confidence, the parameters of the index are kept. The calculation of the confidence level is explained in great detail in the methodology paper; however, the specific confidence level that is applied in the index determination is not given.

Composite indices such as the artnet C50™ Index are adjusted once a year in early January. For the market or sector under consideration the artists are ranked based on the sales data over the last five years. For each year an adapted sales volume V_{kt} is calculated, which corrects for the outliers by taking the median instead of the average,

$$V_{kt} = \text{median}\left(p_{1kt}, p_{2kt}, \ldots, p_{N_{kt}kt}\right) \cdot N_{kt} \, ,$$

where p_{ikt} is the hammer price of the artwork i of artist k in year t and N_{kt} is the number of lots sold of artist k in year t. The rank of a single artist k is calculated by using a weighted sum of the adapted volumes V_{kt}. The weights are exponentially decreasing backwards in time, i.e. the younger sale results have a stronger influence on the rank than the older ones. The exponential factor used by artnet is not published. Once the index is calculated based on the ranking, its starting value for the year is scaled to the closing value of the index composition used during the previous year, i.e. the re-mixture of the artists induces no jumps in the index.

As a collector it is almost impossible to follow or to replicate the index since shifting a collection on a yearly basis into the artists' works with the highest valuation is not feasible. The evidence of such indices is not obvious. Since the indices are *surfing the wave* of the most valuable artists their selection bias is comparable to the survivorship bias for fund indices.[34] See also Salmon (2012) for a critical review of the artnet C50™ Index construction.

artnet has built 135 different comparable sets for Ed Ruscha's works, of which 85 are for paintings that were at least once at auction. Most of the sets contain only a few works, typically around four or five pieces. *Anchor Stuck in Sand* (1990) is covered in the set *Paintings: Silhouettes – Landscape (175x285)* together with *Untitled* (1989) (fig.4). It was sold on 14 November 2001 for USD 620'000 with an

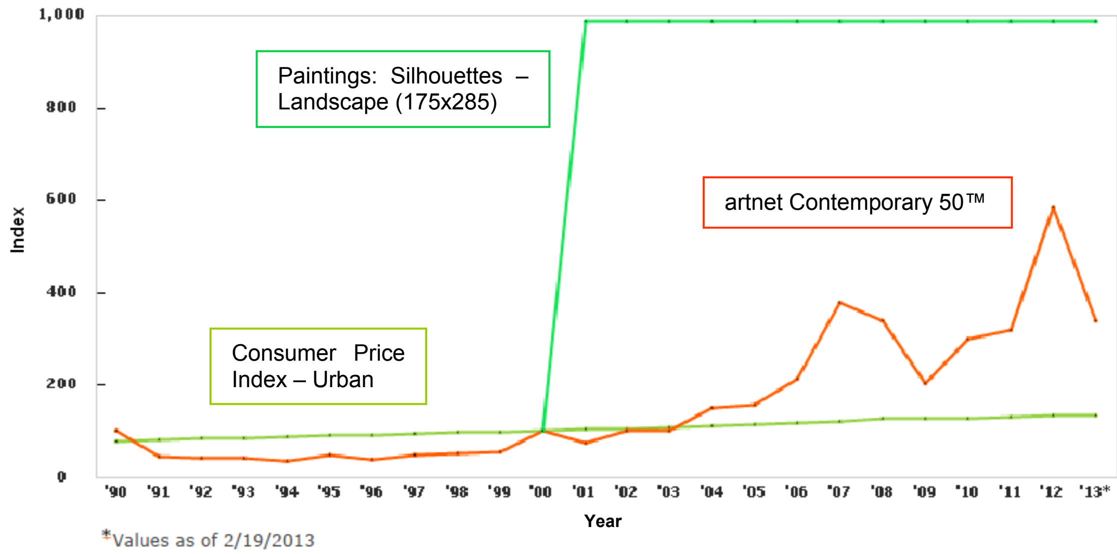
[34] See Elton et al. (1996).

estimate between USD 250'000 and USD 350'000 at Sotheby's in New York. *Untitled* (1989), sold exactly one year after *Anchor Stuck in Sand* by the same auction house, had an estimate that was approximately four times higher and a hammer price that was 10 times higher than for *Anchor Stuck in Sand*. While both works are similar concerning the technique and the size, the subject of *Untitled* (1989) is emblematic for Ed Ruscha's work. He became known for his photographs of gasoline stations, which he later also painted in a stylised form.[35] On the other hand the anchor seems thematically more related to paintings like *Ship Talk* (1988). Therefore, we doubt whether the grouping for *Anchor Stuck in Sand* together with the gasoline station is reliable for a price indication.

Figure 4 Ed Ruscha, *Untitled*, 1989, acrylic on canvas; 152.5 x 284.5 cm, from: http://www.thecityreview.com/f01scra.html, last access on 8.3.2013.

The time series of the index *Paintings: Silhouettes – Landscape (175x285)* is shown in Figure 5. It starts at 100 in 2000, increases to 986 in 2001, and remains unchanged until 2013. From the auction history we can guess that the sharp increase in 2001 is due to the much higher hammer price of the gasoline station compared to the anchor painting, and that the current "estimate" for *Anchor Stuck in Sand* is at *Untitled* 's hammer price of that time, i.e. at USD 620'000 which lies within the estimate of the current auction. However, we suspect that this is a spurious coincidence. First we doubt, as mentioned above, that the two works are comparables, and even if they were, we would have expected at least some volatility in the "estimate" over time.

[35] See e.g. Rowell (2006), p. 96–97, for the original photograph and early drawing studies. The current auction record by Ed Ruscha is at USD 6'200'000 for the early oil painting *Burning Gas Station* (1965–66) sold on 13 November 2007.



*Values as of 2/19/2013
Indices are scaled to earliest year with all components available. Dashed lines represent interpolated values.

Active Indices

Line	Index Name
	Paintings: Silhouettes - Landscape (175x285)
	artnet Contemporary 50™(C50™)
	Consumer Price Index - Urban (CPI-U)

	1990	1991	1992	1993	1994	1995	1996	1997	1998
Paintings: Silhouettes - Landscape (175x285)									
artnet Contemporary 50™(C50™)	99	45	40	41	32	47	38	47	51
Consumer Price Index - Urban (CPI-U)	76	79	81	84	86	88	91	93	95
	1999	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
Paintings: Silhouettes - Landscape (175x285)		100	986	986	986	986	986	986	986
artnet Contemporary 50™(C50™)	54	100	72	101	101	150	155	212	377
Consumer Price Index - Urban (CPI-U)	97	100	103	104	107	110	113	117	120
	2008	**2009**	**2010**	**2011**	**2012**	**2013**			
Paintings: Silhouettes - Landscape (175x285)	986	986	986	986	986	986			
artnet Contemporary 50™(C50™)	338	203	299	317	583	338			
Consumer Price Index - Urban (CPI-U)	125	125	127	131	133	134			

Figure 5 Artnet Report Index Graph for Ed Ruscha's *Paintings: Silhouettes – Landscape (175x285)* which contains *Anchor Stuck in Sand* (1990). The dark green line reflects the index under discussion, the light green line is the Consumer Price Index for All Urban Consumers and the orange line represents the artnet C50™ for Contemporary art. All indices are calibrated to 100 in 2000 when *Anchor Stuck in Sand* has been auctioned. In 2001, after the auction sale of *Untitled* (1989), the index for *Paintings: Silhouettes – Landscape (175x285)* jumps to almost 700 and remains unchanged until 2013. In comparison, the Consumer Price Index and the artnet C50™ Index have increased since 2001 by 26% and 155%, respectively. Table and Chart are from: www.artnet.com, last access on 8.3.2013.

As an example we indicate a Consumer Price Index as a proxy for price inflation in Figure 5, which suggests, everything else being equal, an increase of the prices by 26% since 2001.[36]

Apart from forming the comparable set *Paintings: Silhouettes – Landscape (175x285)* we do not get any additional information for *Anchor Stuck in Sand* by artnet's index building which could not have been obtained from the single auction records; the hedonic price regression is reflected in a rather simplistic manner. Ironically, the grouping of comparables can be viewed on the Internet for free whereas a report with up to ten comparables indices from the same artist is billed.[37]

3.3 Art Market Research

In this section we find that the underlying annual volatility of the AMR Contemporary Art Index in the time period from August 1985 to January 2011 is nearly doubled to almost 29% compared to the reported volatility after applying a de-smoothing methodology.

Art Market Research (AMR) is a private British company, owned and managed by Robin Duthy.[38] The main purpose of AMR is to provide art and collectible price indices.

There is no publicly available description of the index methodology. From various sources the following procedure can be deducted:[39] Auction results are used to build average prices of an artist over a time period of 12 consecutive months. In order to prevent distortion by outliers, the highest and lowest 10% percentiles are omitted, i.e. only the central 80% data is included. Artists are grouped belonging to various schools, movements and periods. The indices are built as equally-weighted portfolios by the corresponding artists on a monthly basis and start mostly in 1974/75. The current index value is determined by the ratio of the portfolio price in the current month to the portfolio price of the corresponding month in the first year of index building.

From a rigorous point of view it is delicate to exclude data due to its position in the sample, and averaged data generally shows a lower volatility than the raw data, i.e. the fluctuation of the prices over time is underestimated.

We consider as an example the index for Contemporary art, which starts in 1985. The artist composition of this index in the year 2002 can be found in Campbell (2008). According to the author, "the choice of artists … is a highly subjective but

[36] Based on logarithmic returns.
[37] For example a single report costs USD 186 (March 2013).
[38] http://www.artmarketresearch.com/default.htm, last access on 2.3.2013.
[39] See Campbell (2008), Ralevski (2008) and Skaterschikov (2010).

representative choice."[40] Our goal is to show the influence averaging has on the volatility on the index. Averaging procedures lead to the effect that the return r_t at time t is related at least to the previous return r_{t-1}, maybe also to older returns. The times series is said to have an *autocorrelation*. We follow Okunev and White (2003) to adjust the reported returns to remove the autocorrelation. The methodology is described in Appendix D.

The monthly data of the AMR Contemporary art index from August 1985 to January 2011 is taken from Bloomberg.[41] This gives 305 monthly returns with a mean return of 0.97% and a standard deviation of 4.23%. We calculate the autocorrelations and remove all of them that lie outside the 95% confidence level band around zero, which is ± 0.112 if we assume normally distributed autocorrelations.[42]

We consider the first 12 orders of autocorrelation and need five iterations to bring them into the required range with the following order autocorrelations removed: 1st, 2nd, 3rd, 6th, and 12th. In Figure 6 we show the first 12 orders of autocorrelation for each of these five return series. The large size of the autocorrelation coefficients reflects the averaging effect used in the price and index building procedure. Note that in the last step we have to correct for a strong negative 12th order autocorrelation. Finally, the mean return becomes 0.77% compared to 0.97% in the original series. This reduction is due to the loss of 24 data points through the consideration of beyond zeroth order autocorrelation.

The monthly standard deviation is almost doubled, from the reported 4.23% to 8.30%. If we assume monthly independence, the monthly volatility can be multiplied by $\sqrt{12}$ in order to get a yearly standard deviation. The respective figures are 14.6% for the reported index and 28.8% for the unsmoothed series. Campbell (2005) finds a difference of 5% between reported and de-smoothed annual standard deviation on average for different indices, a figure that is confirmed in Campbell (2008).[43] The data series of the former publication ends in December 2004 and the latter considers data until February 2006. In our time series we used data until January 2011 and it is not astonishing that the monthly returns since 2006 have induced a lot of volatility. This result is consistent with the findings in Bocart and Hafner (2012). The authors find in a time-varying volatility model a strong increase in the volatility for *blue chip* artists after 2009.

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[40] Campbell (2008), p. 70.
[41] Bloomberg ticker: ARTQCON Index. After January 2011 the data has not been updated anymore and in the meantime the service is completely discontinued on Bloomberg.
[42] See Appendix D.
[43] Campbell (2008), p. 77.

In Table 1 we list the mean, the standard deviation and the first 12 autocorrelation coefficients of the time series as reported, r_{0t}, and of the return series r_{5t}. Before taking the AMR Contemporary Art Index into consideration as a proxy for future potential price development, it is crucial to de-smooth the time series in order not to underestimate the volatility.

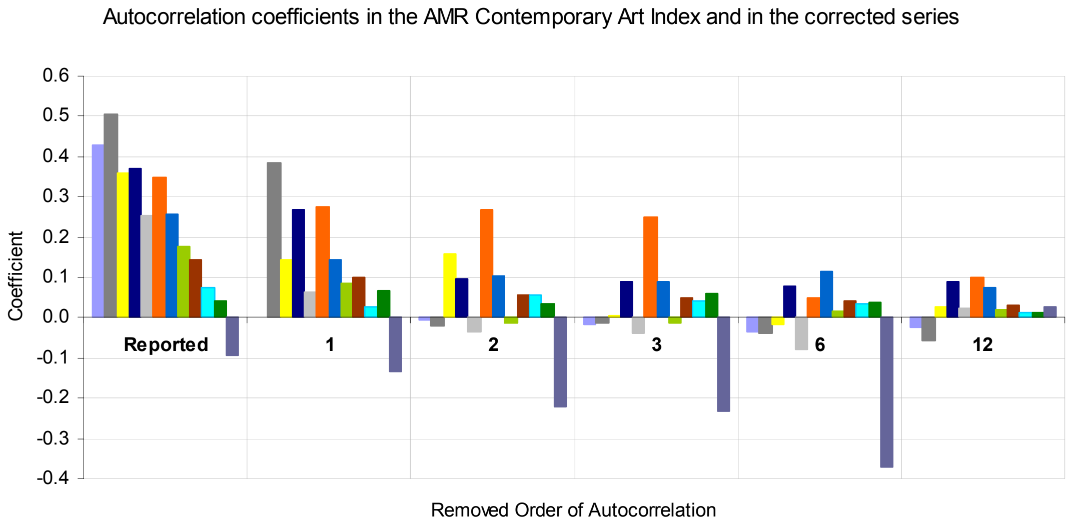

Figure 6 The size of the first 12 orders of autocorrelation (y-axis) is shown for the reported AMR Contemporary Art Index (August 1985 – January 2011) and the five subsequent corrected return series (x-axis). The following order autocorrelations have been removed: 1st, 2nd, 3rd, 6th, and 12th. The 95% confidence level for the normal distribution is at ± 0.112. All autocorrelation coefficients lie within the 95% confidence level in the fifth corrected return series (in the rightmost position). Reported data is derived from Bloomberg, cf. note 41.

Index	mean	standard deviation
r_{0t}	0.97%	4.23%
r_{5t}	0.77%	8.30%

Index	autocorrelation coefficients											
	1	2	3	4	5	6	7	8	9	10	11	12
r_{0t}	*0.430*	*0.505*	*0.358*	*0.371*	*0.252*	*0.348*	*0.257*	*0.177*	*0.145*	0.073	0.043	-0.095
r_{5t}	-0.024	-0.056	0.029	0.088	0.022	0.100	0.073	0.021	0.032	0.012	0.014	0.026

Table 1 The mean, the standard deviation and the first 12 orders of autocorrelation coefficients are shown for the reported AMR Contemporary art index r_{0t} and the de-smoothed return series r_{5t}, respectively. The index 5 refers to a five times corrected time series where the 1st, 2nd, 3rd, 6th, and 12th order autocorrelations have been erased. The 95% confidence level lies at ± 0.11; values outside this band are typed in red italics. Reported data is derived from Bloomberg, cf. note 41.

4 Pricing models

In this Chapter three pricing models are investigated. One is transformed into a hedonic regression and for two others the emphasis is on the price distribution. The need for consistent data from artworks that are as uniform as possible is common to all three applications. Inconsistency in the data can lead to misleading results although statistical tests may be passed.

4.1 Skate's art asset pricing model

In this section we show that Skate's art asset pricing model can be transformed into a hedonic regression with the expected return related to inflation and the idiosyncratic error term to an (unpredictable) irrational premium.

In Skaterschikov (2010) a pricing model is presented which reads

$$P = (FV + IP) \cdot PF \,, \tag{4.1}$$

where P is the price, FV is the fair value of the artwork, IP is an irrational premium and PF is the provenance factor.

The provenance factor PF ranges from zero to one. Zero is associated with a large uncertainty on the authenticity or a doubtful history of the artwork. 1 stands for certainty on authenticity and good provenance. As stated in Skaterschikov (2010), "the PFs for living artists tend to be closer to 1 on average".[44]

The fair value FV is "determined by a peer group comparable valuation method".[45] The peer group examples in Skaterschikov (2010) are limited to similar paintings from a single artist, and essentially the average of the last available prices is taken and corrected for different colours if needed. In the conclusion of the section on fair value the author admits that

> "in practical terms, however, it is rather difficult to build peer groups of comparable works of painting and sculptures. […] Often the best comparable price available is the last known price previously paid for the same artwork (i.e. the original purchase price for a seller), which has been adjusted to reflect the time value of money and ownership costs incurred during the holding period".[46]

In order to correct for the discrepancy between the book or cost value and the possibly higher expected sales price, the irrational premium IP is introduced. The author relates this premium to the artworks marketability, or even more pronounced, to its equity story. Here we are back to the example of Mark Rothko's painting in the

[44] Skaterschikov (2010), p. 77.
[45] Ibid., p. 69.
[46] Ibid., p. 93.

introduction which indeed has been marketed. Again, if it comes to the specifics, the findings are very dry:[47]

> "Any auditlike test of an artwork's value should ignore attempts to quantify the irrational premium, although qualifying the irrational premium at least as 'none', 'possible', 'significant' or 'exceptional' does make sense as a means of differentiating artworks in a collection by their marketability."

When eq.(4.1) is translated to the price equation used in hedonic or repeat-sales regression a transformation is first made of the terms IP to IF, where IF is the irrational factor in order to factorize eq.(4.1),

$$P = FV \cdot IF \cdot PF \text{, and}$$

$$IF = \left(1 + \frac{IP}{FV}\right).$$

This allows for rewriting the price of an artwork i at time t using lowercase letters x, where $x = \ln(X)$,

$$\ln(P_{it}) = p_{it} = fv_{it} + if_{it} + pf_{it}.$$

According to the statement concerning the fair value FV a correction is made for the time value of money. A Consumer Price Index is included as a proxy for the U.S. Dollar's purchasing power, which is assumed to be indexed over time and called CPI_t for further reference. Ownership cost is neglected for this analysis. The log fair value yields therefore

$$fv_{it} = fv_i + cpi_t.$$

There seems to be no common market development in the price evolution apart from the inflation term. Indeed, in the comparison of the present values of the artworks in the database provided in Skaterschikov (2010) the hammer price is indexed by the inflation.[48]

We understand from the statements on the irrational premium cited above that its expectation is zero, and therefore also the expected value of if_{it} vanishes,[49]

$$\mathrm{E}[IP_{it}] = 0 \;\Rightarrow\; \mathrm{E}[if_{it}] = \mathrm{E}\left[\ln\left(1 + \frac{IP_{it}}{FV_{it}}\right)\right] \approx \mathrm{E}\left[\frac{IP_{it}}{FV_{it}}\right] = 0.$$

where $\mathrm{E}[...]$ stands for the expected value. It is also assumed that the provenance factor remains unchanged over time since the quality of ownership history does usually not evolve significantly. Therefore we have

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[47] Ibid., p. 97.
[48] Ibid., p. 240.
[49] See Appendix B.

$$p_{it} = fv_i + cpi_t + if_{it} + pf_i \, .$$

Finally, comparison can be made to the hedonic regression equation eq.(2.1) finding:

$$p_{it}^{Skate} = p_i^{Skate} + p_t^{Skate} + \varepsilon_{it}^{Skate} \text{, with}$$

$$p_i^{Skate} = fv_i + pf_i \, ,$$

$$p_t^{Skate} = cpi_t \, ,$$

$$\varepsilon_{it}^{Skate} = if_{it} \, .$$

The characteristics are given by the fair value of the comparables, often a very restricted sample, and the provenance factor. The price movement in the hedonic regression is associated with the inflation, i.e. art is expected to give no additional returns to the inflation, and the irrational premium is interpreted as an idiosyncratic error term.

The model is rather heuristic in that there is no statistical evidence given and the pricing model is the only formula in Skaterschikov (2010). The validation is provided by argumentation only. These statements are themselves sometimes not without doubts. On the irrational premium the author states:[50]

> "Alas, there is an interesting trade-off to the humanistic, irrational value that is often attached to important art. Art does command a greater residual value in times of distress. When public companies collapse and governments defaults on their debts, the result is often worthless paper. Important artworks, however, maintain a significant residual value. This has to do with the marketability of artworks in any economic climate, as irrational premiums endure, even in times of war and economic decline."

While one may be able to agree on the first part, there are doubts about the second part. Without having studies on the matter at hand, it is hardly imaginable that irrational premiums are sustainable during a war. At least in the Second World War we are not aware of irrationally high prices for artworks.

Skate's Art Market Research provides art finance services. One of the services is the establishment and maintenance of Skate's Top 5000 database, which contains the 5'000 most expensive artworks based on auction prices. After registration this database can be accessed and the data can be exported as a table to Excel, which comes in handy.[51] Furthermore the user has the possibility to build his or her own peer group out of the database, based on the criteria artist, artwork, date of creation, price, date of sale and auction house. The website also contains the repeat-sales

[50] Ibid., p. 100.
[51] Cf. note 2.

within these 5'000 artworks, currently 868 entries, which we will use for analysis in the following Chapter.

4.2 Art pricing in the determination of the expected loss for art backed loans

In this section we show first that statistical tests are no warranty for unbiased auction data and that the price level of art works matters for art backed loans. Secondly it is demonstrated that there exist statistical uncertainties in the positive masterpiece effect of Picasso paintings reported by Scorcu and Zanola (2011).

When a bank gives a loan to a private person it usually makes sure that the person owns some tangible assets that could be transferred to and liquidated by the bank in case of an insolvency of the borrower. This underlying asset is called collateral of the loan and its valuation is crucial in order to make sure that at maturity of the loan the bank gets the full amount back, irrespective of the interest paid by the borrower. One way for the bank to overcome the risk is to transfer it to a third party which would then pay up to an agreed amount in case of the default of the private person. The price to pay for the bank consists of regular premiums to the third party, equivalent to an insurance premium.

Campbell and Wiehenkamp (2009) consider this concept in relation to loans backed by artworks.[52] The authors study the transfer of the potential risk of carrying artworks on the balance sheet from the bank to an unspecified third party. The purpose of the authors is to calculate the worthiness of the art loan taking into account that the debtor can default, or differently formulated, to calculate the expected loss for the art loan.[53] In our analysis the focus is on the price distribution of art works.

Campbell and Wiehenkamp (2009) consider data from Sotheby's London with Impressionist paintings, Victorian pictures, Old Masters, 16th century British paintings and Modern Art. The final data set contains 398 repeat-sale pairs and its empirical return distribution is fitted to normal, logistic and t-Student distributions. The authors find that out of the three statistical functions the t-Student distribution gives the closest description of the data. We reiterate the determination of the parameter of the t-Student distribution function with the repeat-sales data from Skate's Art Market Research. The 868 yearly effective rates of returns are taken and a time-invariant distribution analysis is performed. The range considered starts at -50% and ends at +192%. The histogram shows a peak at around +10% (fig.7). The t-Student

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[52] See also two related publications: Wiehenkamp (2007) and Campbell and Wiehenkamp (2008).
[53] The model is shortly explained in Appendix E.1.

distribution has a mean of 9.9% and a standard deviation of 14.8% with 3.1 degrees of freedom. The high mean will be further investigated below.

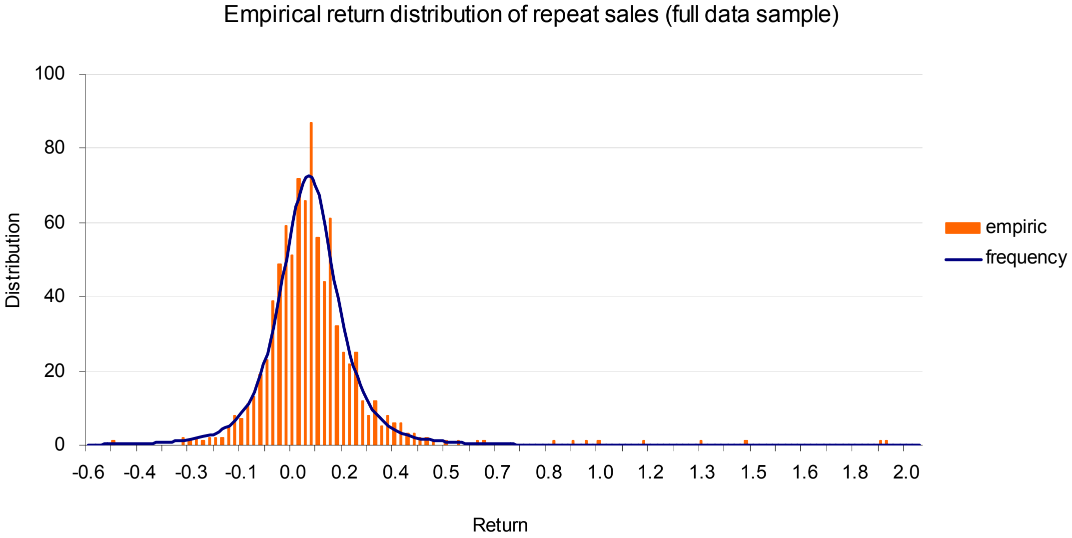

Figure 7 Empirical distribution function of Skate's Art Market Research repeat-sales returns (868 data points; orange bars) and the fitted t-Student distribution function (blue straight line). Data is taken from: www.skatepress.com, last access on 20.1.2013.

The cumulative distribution function is fitted by means of least squares and three tests are performed in order to test the null-hypothesis H_0 that the t-Student distribution is the correct distribution. The tests: Kolmogorov-Smirnov, Cramér-von Mises and Anderson-Darling, are described in Appendix F.

For the tested t-Student distribution function, the confidence level at which the critical value for the rejection of the null-hypothesis is reached corresponds to 56% (Kolmogorov-Smirnov), 55% (Cramér-von Mises) and again 56% (Anderson-Darling). All three tests show the same high confidence level up to which the t-Student distribution should *not* be rejected as correct distribution function for the empirical data. However, the test values should be considered as an indication only for passing the tests since the parameterisation of the function is based on the empirical distribution and is therefore not independently determined.[54] Nevertheless, the test values are high enough to conclude that the t-Student distribution fits well with the data and to confirm what already could have been expected after eye inspection of Figure 7.

A closer look at the dependence of the returns to the initial purchase price is worthwhile. Figure 8 shows the purchase prices in USD on the x-axis and the corresponding yearly returns on the y-axis. The red dots mark the median for the

[54] For a discussion see Campbell and Wiehenkamp (2009).

returns within bands of USD 500’000. The returns are crowded strictly in the positive region for the lower end of the nominals, whereas they are more concentrated near zero return for the larger nominals.

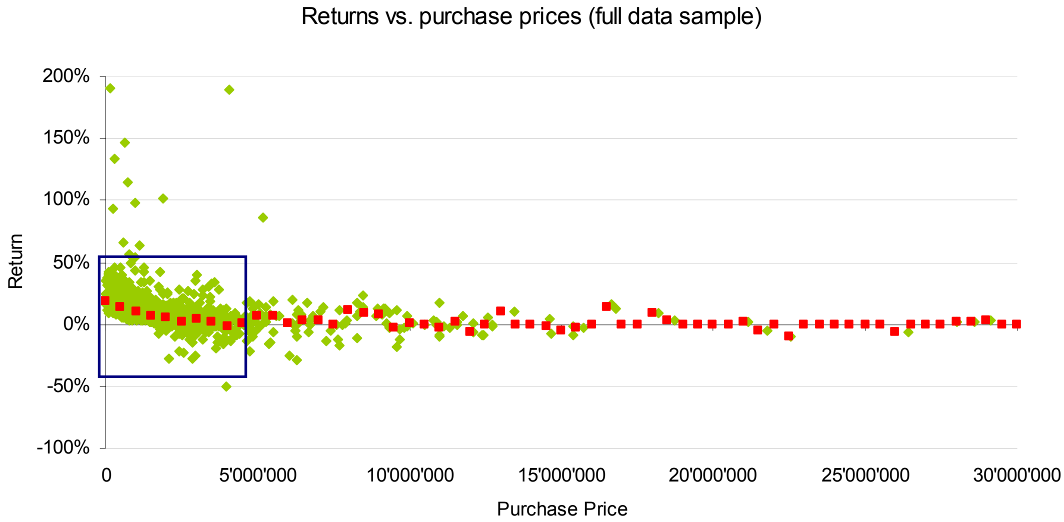

Figure 8 Annual Skate’s Art Market Research returns for repeat-sales (y-axis) vs. the original purchase price in USD (x-axis). The red squares mark the median for the returns within bands for the purchase prices of USD 500’000, and the blue box marks the part which is separately shown in Figure 9. The lower the purchase price the higher the median of the annual return due to the threshold of USD 2.3 million for new entries in Skate’s Top 5000 database.
Data is taken from: www.skatepress.com, last access on 20.1.2013.

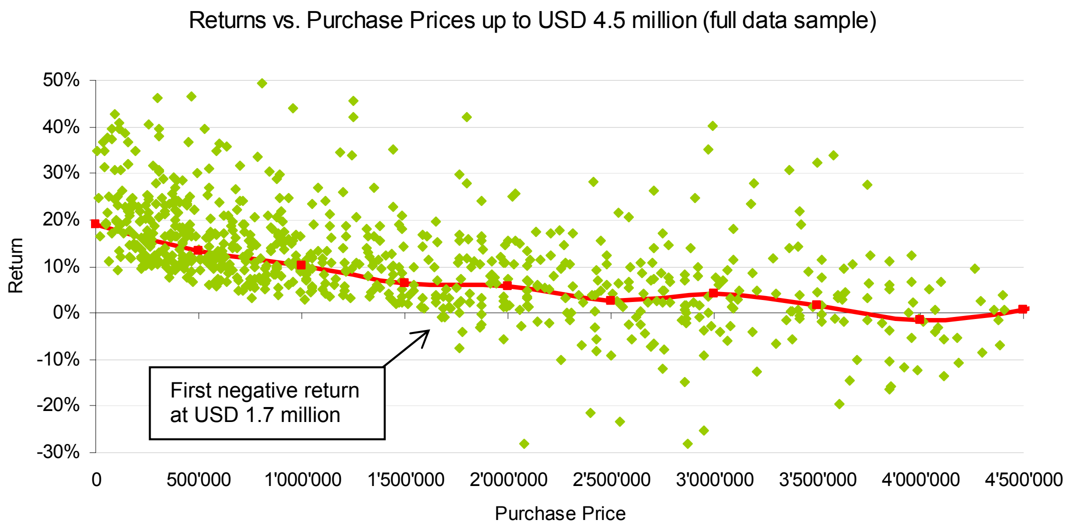

Figure 9 Detail of Figure 8 with the purchase price truncated at USD 4.5 million and at +50% positive and -30% negative annual returns, respectively. The red squares mark the median for the returns bands for the purchase prices of USD 500’000 connected for illustrative purposes with the red line. Data is taken from: www.skatepress.com, last access on 20.1.2013.

In fact, as can be seen from Figure 9, there are no negative returns up to an original purchase price of USD 1.7 million. Since the lowest entry of Skate's Top 5000 database has a present value of USD 2.3 million, an artwork with a single purchase price below this threshold and a negative return rate at the resale falls out of the sample. Therefore the sample at hand is truncated for smaller purchase prices, and the distribution found to fit the empirical data quite well does not consider the truncated returns. Hence it is not astonishing that the t-Student distribution has a high mean of almost 10%. The sample therefore is reduced to make sure that potential losses of -50% would still be above the threshold of USD 2.3 million and would not disappear from the data sample, and repeat the fit procedure.

The adaptation of the data sample leads to the constraint that all data below USD 4.5 million is neglected and we are left with 129 resale returns. The t-Student distribution is still a good estimate, although it is less obvious from Figure 10.

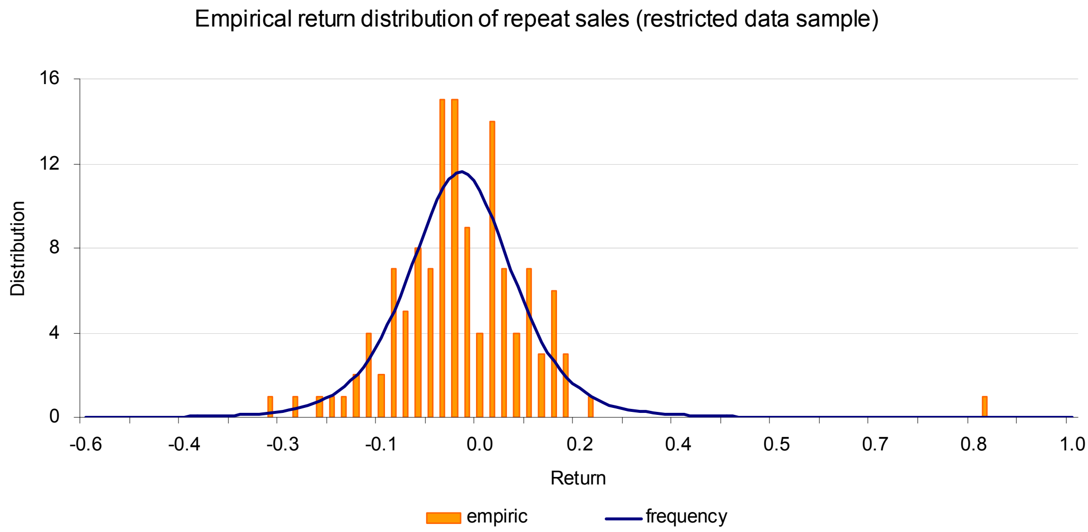

Figure 10 Restricted empirical distribution function of Skate's Art Market Research repeat-sales return for purchase prices above USD 4.5 million (129 data points; orange bars) and the fitted t-Student distribution function (blue straight line).
Data is taken from: www.skatepress.com, last access on 20.1.2013.

With the restricted sample the mean and the standard deviation are significantly lower at 1.9% and 10.8%, respectively. The degrees of freedom ν for the restricted sample are at 5.5, somewhat larger than for the full sample, i.e. it has slightly lower weights in the tails, which indicates that the restricted sample is closer to the normal distribution, although still substantially different.[55] The larger number of degrees of

[55] The normal distribution is considered to be a good proxy for the t-Student distribution in case of $\nu > 30$.

freedom is also the reason for the lower standard deviation, since the scale parameter remains almost unchanged at 8.6% for the restricted sample, compared to 8.8% for the complete data set. With $\nu > 4$ it is possible to calculate the excess kurtosis $\gamma_2 = 1.33$, which is also a sign that the distribution is fatter in the tail than a normal distribution with vanishing excess kurtosis. Table 2 summarises the parameter estimates of the t-Student distributions for the two data samples.

Parameter estimates	**Full sample**	**Purchase price > USD 4.5 million**
location μ	9.9%	1.9%
scale σ	8.8%	8.6%
degrees of freedom ν	3.10	5.52
mean $\mathrm{E}[X] = \mu$	9.9%	1.9%
standard deviation $\sqrt{\mathrm{var}(X)} = \sigma \cdot \sqrt{\frac{\nu}{\nu - 2}}$ for $\nu > 2$	14.8%	10.8%
excess kurtosis $\gamma_2(X) = \frac{6}{\nu - 4}$ for $\nu > 4$	n.a.	1.33

Table 2 Statistics for the fitted t-Student distribution to the empirical distribution function of Skate's Art Market Research repeat-sales returns. The middle column shows the figures for the full sample (868 data points) and the right column contains the results for the restricted sample with a minimal purchase price of USD 4.5 million (129 data points). The excess kurtosis is only available for the restricted data sample due to the degrees of freedom.

The test results are now different. The Kolmogorov-Smirnov test gives no restriction since the point-wise distances between the empirical data and the cumulative distribution function are too small to find a corresponding critical confidence level. The Cramér-von Mises test is also not very restrictive, the confidence level is found to be 96%, i.e. the distribution is not rejected up to this level.

The Anderson-Darling test on the other hand is the most restrictive one with a p-value of 25%. Based on the discussion in Appendix F of the weight function for the squared differences, we can conclude that the assumed distribution function seems to work well around the mean, whereas it is only restricted applicable in the tails. All the test results are listed in Table 3 for both samples under investigation.

Parameter estimates	Full sample		Purchase price > USD 4.5 million	
	Value	corresponds to c.I. for rejection	Value	corresponds to c.I. for rejection
Kolmogorov-Smirnov $d_{\max}$	0.0271	56%	0.0448	n.a.
Cramér-von Mises W_n^2	0.1081	55%	0.0337	96%
Anderson-Darling A_n^2	0.5556	56%	0.2494	25%

Table 3 Test statistics for the fitted t-Student distribution to the empirical distribution function of Skate's Art Market Research repeat-sales returns. The third column shows the figures for the full sample (868 data points) and the rightmost column contains the results for the restricted sample with a minimal purchase price of USD 4.5 million (129 data points). The Kolmogorov-Smirnov test is not applicable in case of the restricted data sample since the test value is too low. However, the test values should be considered as an indication only for passing the tests since the parameterisation of the t-Student distribution is based on the empirical distribution and is therefore not independently determined.

In Campbell and Wiehenkamp (2009) the fitted t-Student distribution should be rejected at the 5% confidence level according to the Cramér-von Mises test.[56] Although the Anderson-Darling test is explained, unfortunately the values are not reported. From the QQ-plot in the extended version Wiehenkamp (2007) on the same subject, it can be seen that the fit with the t-Student distribution works quite well around the mean, but that there are a few strong outliers in the tails. Since the Anderson-Darling test puts more weight in the tails, it can be guessed that the test would possibly have failed.

Campbell and Wiehenkamp (2009) report a mean of 3.2%, a scale parameter of 5.6% and degrees of freedom $\nu = 2.12$, which gives a standard deviation of 24.0%. The mean and standard deviation that we found for the sample with a purchase price above USD 4.5 million are substantially lower than those in Campbell and Wiehenkamp (2009) which can lead to two different conclusions: Either the returns found in both studies are not universal, i.e. the return distributions are *not* applicable to every kind of underlying, meaning that it matters whether we take Impressionist paintings as in Campbell and Wiehenkamp (2009), or all kinds of artworks, but highly priced, as provided by Skate's Art Market Research. Or, the lower mean return found above is due to the price level, also called *Underperformance of Masterpieces*: Mei and Moses (2002) found that the future excess price of masterpieces is lower

[56] Campbell and Wiehenkamp (2009), p. 26.

compared to other price categories.[57] A recent update is provided in Deloitte (2011) based on the Mei Moses World Art Index which shows declining returns with increasing purchase prices.[58] It is noteworthy that the data sample has been truncated since returns above 300% have been excluded.

Different findings concerning this topic can be found in Renneboog and Spaenjers (2009) and Scorcu and Zanola (2011). Renneboog and Spaenjers (2009) found a higher performance for masterpieces based on a large sample of auction results. Scorcu and Zanola (2011) did a specific study on the different price level returns for Picasso paintings. Picasso is actually by far the “most valuable artist” in Skate’s Top 5000 database and the amount of valuable works on the art market is large.[59] The authors reported a higher return for higher priced paintings, accompanied by a higher volatility. They use a *quantile hedonic regression,* which means that they build weighted sums of the squared residuals in order to fit the empiric data. The weights depend asymmetrically on the distance from a specific quantile, for details we refer to Scorcu and Zanola (2011). The authors choose the quantiles 0.2, 0.4, 0.6, 0.8 and 0.95. For each of them a yearly index is established. If there are any systematic differences, a constant term should be found that is different from zero in a regression between the different quantile return series. The return series of the quantile j is a linear regression of the return series of the quantile i,

$$r_{jt} = \alpha_{ij} + \beta_{ij} \cdot r_{it} + \varepsilon_{ijt} . \tag{4.2}$$

Figure 11 shows the α_{ij} for each pair of return series including the one standard deviation, and put all $\beta_{ij} = 1$.[60] We observe that the 0.95-quantile has an increased expected return compared to the other quantile return series, but the statistical evidence is too weak.

Scorcu and Zanola (2011) “develop a more balanced approach” concerning the asynchronicity of the return cycles and use in addition a five year rolling mechanism.[61] Indeed, the repetition of the above linear regression gives systematically higher α_{ij} for the 0.95-quantile, with t-statistics between 1.44 and 2.03 (fig.12). However, the returns in this series are not independent from each other due to the rolling mechanism.

[57] In this case masterpieces are only defined by their price.
[58] Deloitte (2011), p. 20.
[59] By year end 2012, the total value of the transactions related to Pablo Picasso’s works included in Skate’s Top 5000 database is USD 3’252 million, whereas the next entry is Andy Warhol with an overall transaction value of USD 1’703 million, cf. note 19, p. 4.
[60] Allowing for different β_{ij} does not change the findings.
[61] Scorcu and Zanola (2011), p. 96.

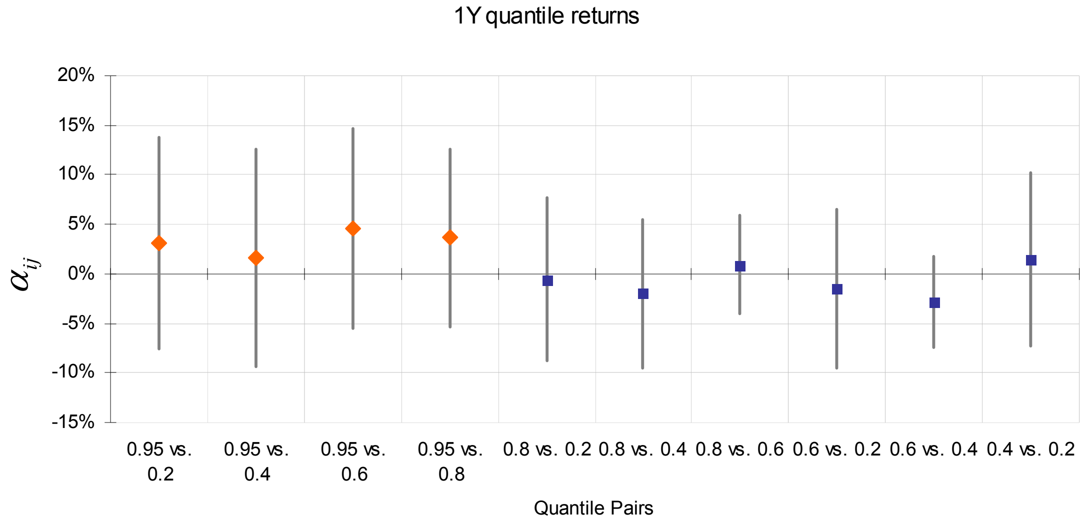

Figure 11 Constant term α_{ij} (y-axis) in the linear regression eq.(4.2) with $\beta_{ij} = 1$ between pairs of annual return series (x-axis) based on quantile hedonic regressions for Picasso paintings. The bars reflect the one standard deviation and orange diamonds refer to combinations where the 0.95-quantile is considered. None of the terms α_{ij} deviates significantly from zero. Index data is taken from: Scorcu and Zanola (2011).

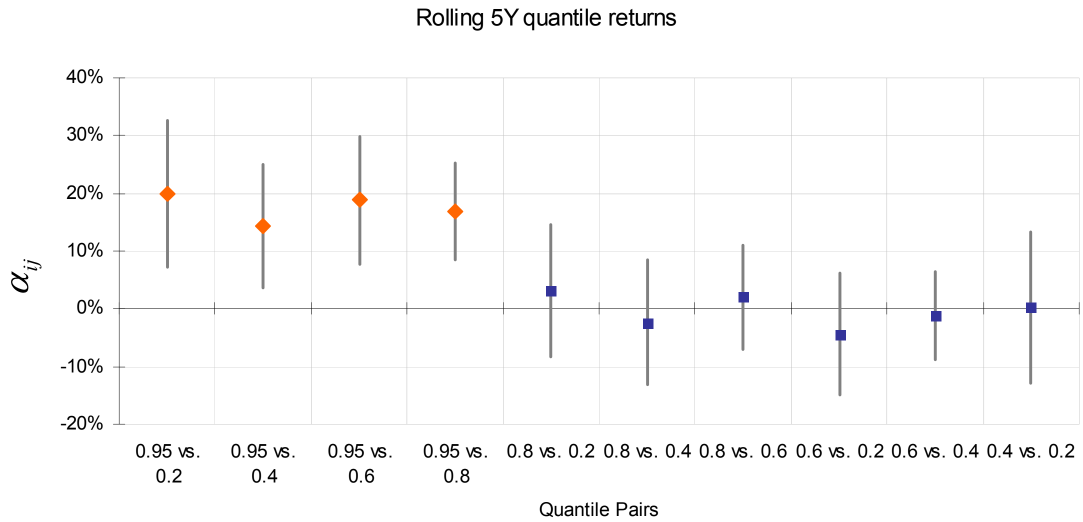

Figure 12 Constant term α_{ij} (y-axis) in the linear regression eq.(4.2) with $\beta_{ij} = 1$ between pairs of rolling five year returns series based on quantile hedonic regressions for Picasso paintings. The bars reflect the one standard deviation. Orange diamonds refer to combinations where the 0.95-quantile is considered, all of which deviate positively from zero at the one-sigma level. Index data is taken from: Scorcu and Zanola (2011).

Finally, we show that the volatility is a *decreasing* function for the logarithm return of the rolling five-year return, contrary to the statement in Scorcu and Zanola (2011). The authors' different finding, graphically represented in their Exhibit 8, is most probably due the representation of the return as the change in the index scaled in each starting year to 100, called *index return*. Figure 13 shows the volatilities compared to the returns for the one-year and the rolling five-year return series of the full sample and the quantile return series.

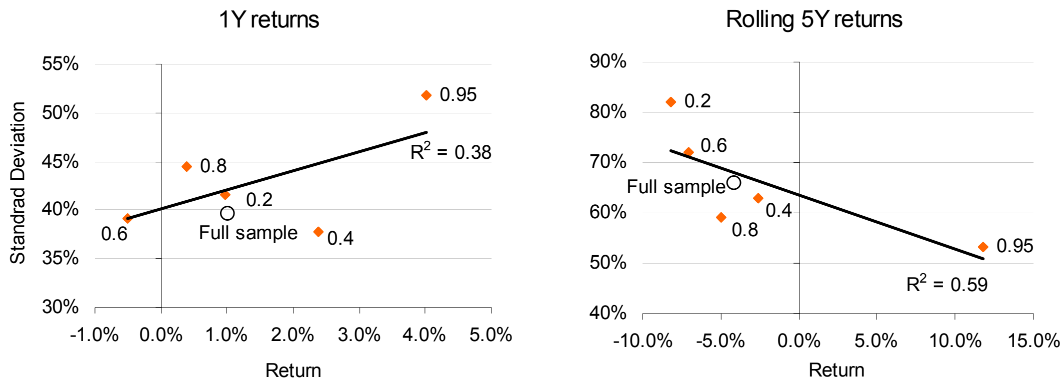

Figure 13 Standard deviations (y-axis) compared to logarithmic returns (x-axis) for the annual and the rolling five year return series of the full sample (circle) and the quantile return series (orange diamonds). The straight line reflects the linear regression of the different ratios. Index data is taken from: Scorcu and Zanola (2011).

To conclude regarding Scorcu and Zanola (2011), we see some evidence that Picasso's masterpieces have a higher performance than the less expensive works. However, due to the statistical uncertainty for the one-year returns and the moving-average-like procedure used to proove the overperformance of the high valued works, we abandon the conclusion that Picasso's masterpieces do indeed have a higher performance.

In any case, the price distribution in Campbell and Wiehenkamp (2009) is assumed to be universal for the calculation of future potential losses related to loans. Although the authors state that their data set does not allow for a split in sub-sets, we recommend to reconsider this assumption and to use price level dependent parameterised price distributions. This differentiation can be especially relevant if the artworks are considered for loans and therefore are subject to expected loss considerations.

Also, it has to be kept in mind that bought-ins are excluded from the beginning, which is a topic that will be treated in the following Chapter.

4.3 Artworks as collateral values in bank loans

In this section it is found that the attempt of McAndrew and Thompson (2007) to model auction results including bought-ins can be applied to Ed Ruscha's works and we present an explicit "price" distribution of the bought-ins recorded for Ed Ruscha in relation to the pre-sale estimates.

With respect to the value of artworks for bank loans McAndrew and Thompson (2007) consider auction prices of French Impressionist paintings in relation to the pre-sale estimates. The authors explicitly model the bought-ins under the assumption of specific distribution features and calculate the loss potential of an artwork in case it is used as collateral for a bank loan. In order to illustrate the mechanism of their approach the calculation is repeated with the American contemporary artist Ed Ruscha. Although he worked with diverse media like oil paintings, paper collages, prints, photographs and films, he has quite a homogeneous portfolio in the sense that the subjects and themes that he covers in his artworks remained the same since the start of his artistic career. This homogeneity makes him a preferred candidate for the application of a general pricing model.

4.3.1 Ed Ruscha

Edward Joseph Ruscha was born in Omaha, Nebraska in 1937 and grew up in Oklahoma City.[62] He moved to Los Angeles in order to study advertising design at the Chouinard Art Institute from 1956 to 1960. He still lives and works in California.

As a child Ruscha was fascinated by comics and as a student worked in a print shop where he learned manually type setting. These anecdotal events can be recognised later on in his artistic work. At the beginning of the 1960's he started to produce photo books from everyday environmental elements: gasoline stations, parking lots, swimming pools and baby cakes. The photographs were taken intentionally to appear amateur-like and they were empty with respect to people, with the exception of his son as a baby. The interpretation is still debated, but the influence of his work on other conceptual artists is indisputable.[63]

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[62] Sources for this part are Marzona (2005), p. 90, Art + Text (2006), and Encz (2009), p. 19–27.

[63] Edwards (2004), p. 143, states that "it is not clear if the viewer is supposed to concentrate on the photographs or on the objects represented; […] Looking at the [photo]book is so 'perplexing' because it involves contemplating an everyday form removed from its everyday use." Iversen (2010), p. 24, says that "the condition under which photography was acceptable as a medium for Ruscha, and for a number of other artists of this generation, was as a performative act executed in accordance with a set of instructions or simple brief. […] The text or title in Ruscha's practice is a general, fairly empty or abstract instruction, while the photographs represent specific instances or realisation[s]."

In parallel he made paintings that were, according to Iversen (2010), more easily interpretable in the context of Pop Art and Jasper Johns than the photographs. In the paintings he again depicts every day objects like the gasoline stations. Often he combines the bright colour paintings with text, mostly short simple expressions or single words. Consequently, he also paints words alone on a coloured background. According to Art + Text (2006), “Ruscha sees words as still-lifes and his paintings as the fictitious titles of books.”

He had his first solo show at the Ferus Gallery, Los Angeles in 1963 and is currently represented by the Gagosian Gallery, where his first exhibition took place in 1982.[64] Beside many solo exhibitions in museums, Ruscha was the representative of the USA at the 51st Venice Biennale in 2005.

4.3.2 Data Selection

Auction results of Ed Ruscha’s work were retrieved from Artprice on 26 and 27 December 2012. Drawings, paintings and photographs are considered. Prints are not taken into account in order to concentrate on unique works. However, in spite of the possibility of multiple prints, the photographs are included because they are regarded as being closer to paintings than to prints or multiples in the case of Ed Ruscha.

The data was typed manually from the screen since Artprice did not provide raw data from their auction database. Data corrections were done with care and only when absolutely sure about the error.[65]

In total 731 entries were retrieved: 176 drawings, 473 paintings and 82 photographs. 19 entries were eliminated due to quality doubts:

- One entry due to a unrealistic low estimate,
- 18 sold works due to uncertainties whether the prices shown are hammer prices (as indicated) or whether they include the buyer’s premium.

Next, 15 entries were dropped that did not have pre-sale estimates since the approach used below is based on pre-sale estimates. Furthermore, seven entries were neglected with a point-estimate only, i.e. with missing information on the lower estimate, which is necessary in the fitting approach.

Finally 690 valid auction records are found, which are shown per category in Table 4. We also indicate the separation between sold works and bought-ins, and the sold works with a hammer ratio above a specific threshold, to be explained in the next Chapter, and the bought-in ratio compared to the final set.

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[64] http://www.gagosian.com/artists/ed-ruscha, last access on 21.3.2013.
[65] At the time of data collection all currencies except Euro and US Dollars had been temporarily misspecified by Artprice as British Pounds. Positions were manually adjusted after Artprice had again corrected the corresponding entries. In addition, one doubtful estimate could be checked and corrected with the help of Christie’s online catalogue.

Ed Ruscha's auction results	**Total**	**Drawings**	**Paintings**	**Photographs**
Initial set	**731**	**176**	**473**	**82**
- Quality issues	19	5	14	0
- No pre-sale estimate	15	4	9	2
- Point-estimate	7	0	2	5
Final set	**690**	**167**	**448**	**75**
thereof sold works bought-ins	558 132	140 27	366 82	52 23
Sold works with hammer ratio above threshold t_b	*391*	*99*	*253*	*39*
Bought-in ratio of final set	*19%*	*16%*	*18%*	*31%*

Table 4 Ed Ruscha's auction results for drawings, paintings and photographs until 30.11.2012. The second column contains the sum of the figures in the columns three to five. The second row shows the initial data set that has been retrieved from Artprice on 26 and 27 December 2012. The rows three to five contain the numbers of excluded records due to quality issues, missing pre-sale estimates and no separation of lower and upper estimates (point-estimate), respectively. The sixth row (bold) represents the final data set that is used in the analysis. This set is further split into sold works and bought-ins (seventh row). The sold works with a hammer ratio $h = P / FV$ above the threshold $t_b = 0.857$ are given in the eighth row. P is the hammer price and FV the pre-sale geometric mean of upper and lower estimate U and L, respectively. This specific data set is used for the determination of the hammer ratio distribution. The last row shows the ratio of the bought-ins compared to all auction records considered in the final data set.
Data is taken from: www.artprice.com, last access on 27.12.2012.

4.3.2 Application

The approach by McAndrew and Thompson (2007) considers the loss in case the borrower is unable to pay back the bank loan (or pay the interest). The authors assume that the valuation of the artwork can be done based on the auction history of a single artist, the period for different artists or other criteria leading to comparable art objects. The method is briefly described in Appendix E.2.

The approach is based on the assumption that the expert in an auction house knows the fair value FV of the artwork under consideration. The hammer price P is put in relation to FV as the hammer ratio h,

$$h = \frac{P}{FV}.$$

The authors model the pre-sale estimate as if an unbiased auction expert first establishes the expected hammer price FV, and then selects an upper and lower estimate U and L, respectively, such that the geometric mean of U and L equals FV and the spread between U and L reflects the experts' uncertainty on the expected price. Therefore:

$$h = \frac{P}{\sqrt{U \cdot L}}.$$

In case the auction lot is a bought-in, the result is temporarily not considered in the analysis, rather it is only noted that there was a bought-in. The incorporation of bought-ins takes place at a later stage.

In Figure 14 the distribution of the hammer ratios of the auction results of Ed Ruscha's works sold at auctions is shown including the 558 data points that were selected according to the procedure explained above.

The mean hammer ratio lies at 1.21, the largest value is at 5.54 and the smallest value at 0.33. This large spread between maximum and minimum leads to a standard deviation of 0.63. The mean is substantially increased by the outliers at the upper end; the median is with 1.04 much closer to the geometric mean of the pre-sale estimates, i.e. in case a work of Ed Ruscha is sold at auction, roughly 50% of the results lie below the expected hammer price.

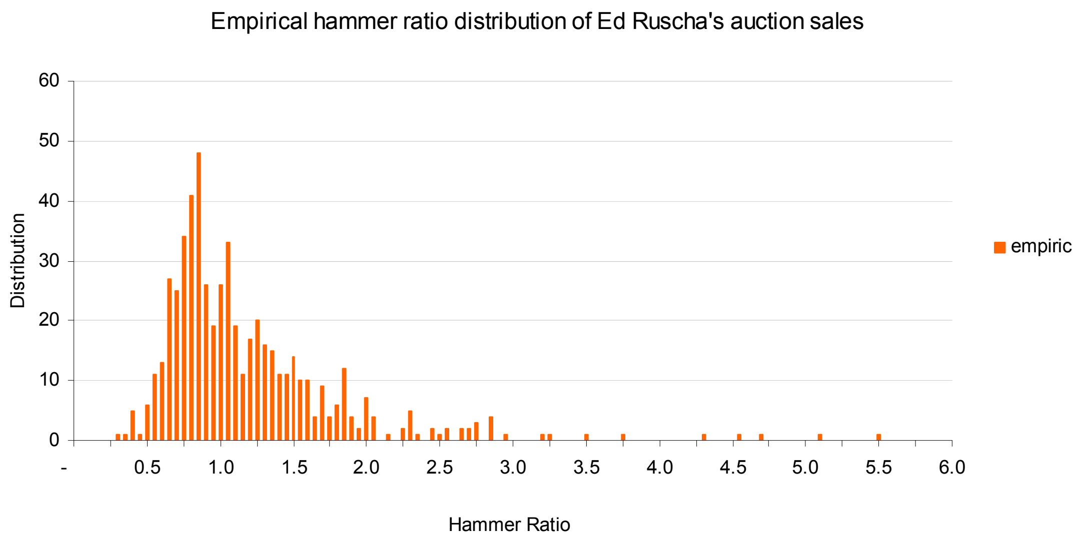

Figure 14 Empirical distribution of the hammer ratios $h = P/FV$ for Ed Ruscha's auction sales for drawings, paintings and photographs until 30.11.2012. P is the hammer price and FV the pre-sale geometric mean of upper and lower estimate U and L, respectively. The chart shows the sold works in the final set in Table 4 (558 data points).
Data is taken from: www.artprice.com, last access on 27.12.2012.

The goal is to find a distribution function of the auction results, which potentially incorporates the bought-ins. McAndrew and Thompson (2007) assume a lognormal distribution for their data sample. The reason for the log normality is given on a qualitative level, supported by an “optical” coincidence of a fitted distribution to the empirical data and a Kolmogorov-Smirnov test. In order to parameterise the function, the authors start with the observation that the bought-ins, left out so far, can only occur in the left tail of the distribution, since the highest bid, in case of bought-ins, does not exceed the potentially existing reserve price. And the reserve price is not allowed to exceed the lower pre-sale estimate L. It is therefore almost certain that the virtual price of a bought-in b is lower than its lower pre-sale estimate L_b. In addition we made sure with the data selection that lower and upper estimates exist,which are different from each other, and thus for a bought-in b,

$$l_b = \frac{L_b}{FV_b} < 1.$$

For later use we define a threshold t_b as the maximum of all l_b,

$t_b = \max\left(l_{b_1},..,l_{b_n}\right)$, for all bought-ins b_i.

Hence, the right tail of the distribution shown in Figure 11 is complete with respect to the auction results considered. When moving to the left on the scale of the hammer ratio, point $h < 1$ is reached, where the highest bid of a bought-in could enter the ratio distribution. This point corresponds to t_b; in the case at hand, this parameter is at 0.857.

McAndrew and Thompson (2007) aim to fit the distribution in the region of the right tail where the data is complete, i.e. for hammer ratios larger than t_b, with a technique that was developed by Cohen (1959). This approach considers a normal distribution that is truncated or censored in one of the tails and yields the mean and standard deviation of the complete distribution by means of a functional form.[66] The needed input consists of the sample data above or below the threshold where the distribution is truncated, and of the number of the dismissed data points.

In our case the truncation takes part in the left tail, which contains all the bought-ins with their unknown bids. We deviate from the procedure in McAndrew and Thompson (2007) insofar as they take the average of the low estimate to the geometric mean pre-sale estimate of the works sold as a proxy for the threshold in the fit procedure, instead of the above defined t_b. Content wise no correspondence

[66] See Appendix G for a description of the method.

exist between the two figures. However, numerically they are very close to each other in the case of Ed Ruscha's auction results, the former being at 0.847, compared to the latter at 0.857.

The data set of hammer ratios for sold works below 0.857 (167 data points) is temporarily removed leaving 391 auction records for the determination of the truncated distribution.

Finally this results in a mean μ of the normal distribution of -8.0% and a standard deviation σ of 52.8%. This translates into a mean and a standard deviation for the lognormal distribution via

$$\mathrm{E}[X] = \exp\left(\mu + \frac{\sigma^2}{2}\right),$$

$$\sqrt{\mathrm{var}(X)} = \mathrm{E}[X] \cdot \sqrt{\exp(\sigma^2) - 1},$$

and gives a mean of 1.061 and a standard deviation of 0.602. This mean is substantially lower than in the case where only sold works have been considered (1.21, see above) and much closer to the median in that case (1.04). The standard deviation is comparable to the one from the empiric distribution for sold works (0.63). Thus, the inclusion of the bought-ins within the framework of Cohen (1959) reduces the mean significantly and leaves the standard deviation almost unchanged. Here the analysis of Campbell and Wiehenkamp (2007) is concluded with respect to the hammer ratio distribution.

In order to check the results, we estimated the empirical data directly to a lognormal distribution by an ordinary least square estimator above the threshold t_b, resulting in a location and scale parameter of -9.5% and 53.0%, respectively. This gives a mean of 1.046 and a standard deviation of 0.596, close to the corresponding parameters of the former approach. The Kolmogorov-Smirnov test is performed on the sample that we fitted, i.e. above the threshold t_b. The test gives a very high confidence level of 93% for the null-hypothesis, that the lognormal distribution should not be rejected. However, the same restrictions apply as in Chapter 4.2. The parameters are found based on the empirical data and the test is therefore not unbiased. Moreover, since only the upper tail is considered, potentially higher differences between the data and the assumed distribution below the threshold is not taken into account.

Figure 15 shows the results due to the determination of the lognormal distribution. Highlighted in orange are the data points above the threshold t_b that have been considered in the determination of the distribution. The red line represents the density

function related to the approach of Cohen (1959) and the blue line is the density function found by the fitted procedure described above. Both lines are by eye inspection hardly distinguishable, the only visibly detectable difference between the results of the two approaches lies in the region below the threshold.
The next step consists in adding the missing data points. The green bars represent the data of works sold with a hammer ratio below the threshold t_b. For the bought-ins we us the lognormal density function found by Cohen's approach as a guidance. Since the sold works below the threshold exceed the assumed function around 0.75, the bought-ins do not completely fill the remaining area of the density function. Therefore, above a (virtual) hammer ratio of 0.5 we assume the bought-ins to come next to the assumed function, whereas below 0.5 we allow for a deviation from the found density. The data below 0.5 are assumed to follow an arbitrary lognormal distribution with the constraints that a) the data points above 0.5 including the bought-ins and the works sold below the threshold t_b fit also to this arbitrary lognormal distribution, and that b) the total number of bought-ins, i.e. 132 data points, has to be respected.

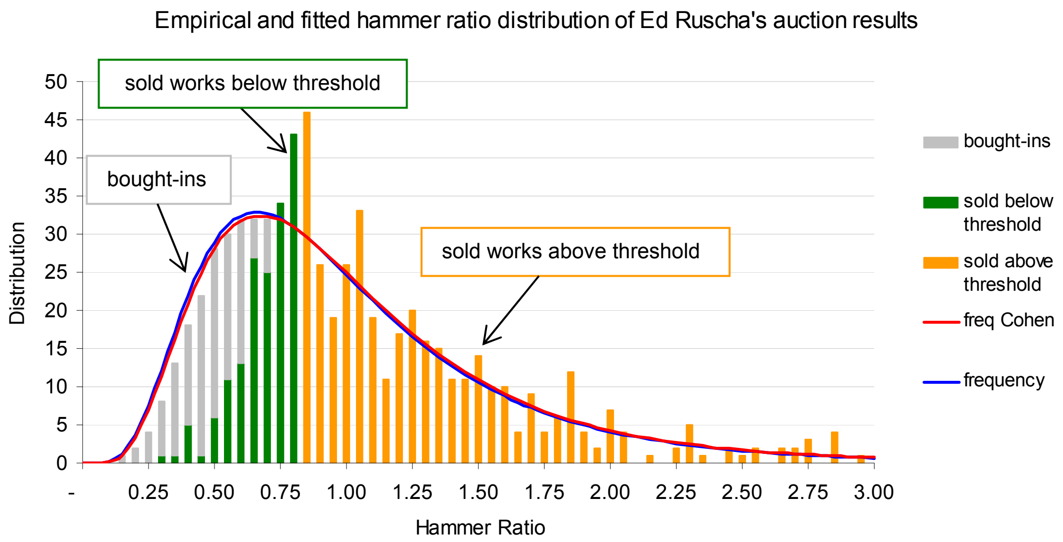

Figure 15 Empirical distribution of the (virtual) hammer ratios $h = P / FV$ for Ed Ruscha's auction results for drawings, paintings and photographs until 30.11.2012 including an assumed distribution for bought-ins (in total 690 data points; corresponding to the final set in Table 4). P is the (virtual) hammer price and FV the pre-sale geometric mean of upper and lower estimate U and L, respectively. The orange bars, representing 391 data points, are sold works above a hammer ratio threshold $t_b = 0.857$ and are used to find the distributions shown as red and blue lines, respectively. The green bars are sold works below t_b (167 data points) and the grey bars are the bought-ins (132 data points) that are shown separately in Figure 16. Data is taken from: www.artprice.com, last access on 27.12.2012.

The resulting distribution for the bought-ins is shown as grey bars in Figure 15. The choice of 0.5 as the point where the bought-ins are allowed to deviate from the lognormal distribution valid for higher hammer ratios is based on the assumption that the highest bid for a lot which does not reach half of the pre-sale estimate occurs very rarely. Rather, the bought-ins are expected to be above 50% of the assumed fair value. Otherwise the auction expert would have overestimated the artwork by 100%!

The stand-alone distribution for bought-ins can be associated with a normal distribution of a mean 0.51 and a standard deviation of 0.12. In Figure 16 the bought-in data is shown as grey bars and the line represents the fitted normal distribution. The largest values for bought-ins are around 0.7 which is in-line with the results found by McAndrew et al. (2009) and Ashenfelter and Graddy (2011) on the size of the reserve price.

McAndrew et al. (2009) derived an expected reserve price of 0.648 for the bought-ins in terms of the geometric mean pre-sale estimate. The authors use the same data as in McAndrew and Thompson (2007). From their Figure 4 it can be estimated that their mean of the bought-ins is around 0.5 and that bought-ins do not exceed 0.78.

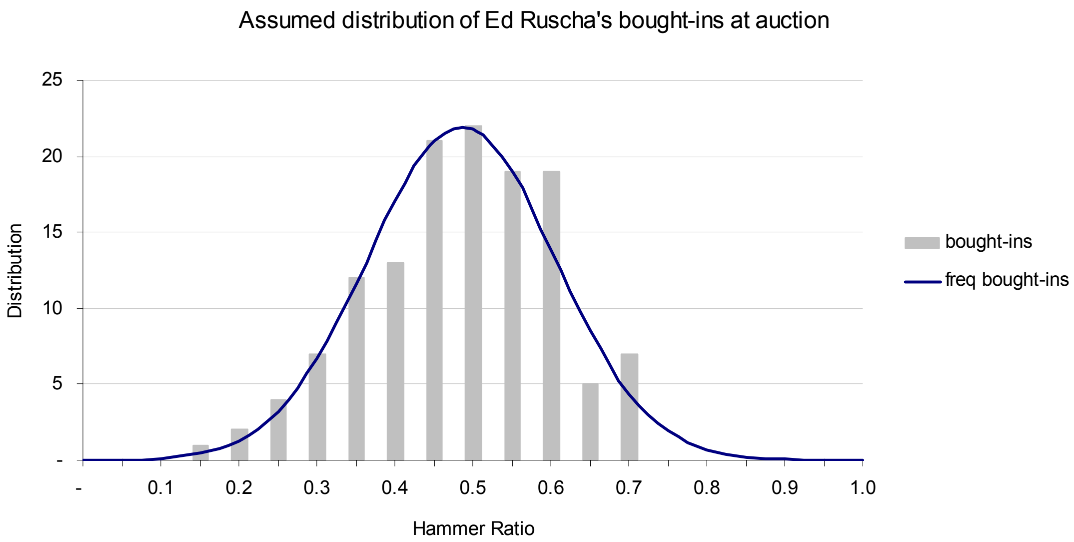

Figure 16 Assumed normal distribution (dark blue line) of Ed Ruscha's bought-ins (grey bars; 132 data points) for drawings, paintings and photographs until 30.11.2012 in relation to FV, where FV is the pre-sale geometric mean of upper and lower estimate U and L, respectively. The grey bars show virtual hammer ratios $h = P / FV$ that are derived from the distribution function shown in Figure 15.

Ashenfelter and Graddy (2011) found that the reserve price is on average 0.71 of the low estimate. In order to translate their result into a ratio of the geometric mean pre-sale estimate, their figure is multiplied with the average ratio of the lower to the geometric mean pre-sale estimate from this study, $L/FV = 0.85$. The result is 0.60, which coincides nicely with the above mentioned finding in McAndrew et al. (2009).

Listed in Table 5 are all the parameter results obtained in the current Chapter, concluding that the model of McAndrew and Thompson (2007) is applicable to Ed Ruscha's auction results for drawings, paintings and photographs. A distribution for the bought-ins is determined, which was not found in this explicit form in the literature.

Parameter estimates	**Cohen**	**Direct approach**
Log normal distribution		
Location μ	-8.0%	-9.5%
scale σ	58.8%	53.0%
mean $\mathrm{E}[X] = \exp\left(\mu + \frac{\sigma^2}{2}\right)$	1.061	1.046
standard deviation $\sqrt{\mathrm{var}(X)} = \mathrm{E}[X] \cdot \sqrt{\exp(\sigma^2) - 1}$	0.602	0.596
Kolmogorov-Smirnov $d_{\max}$	n.a.	0.0236
Kolmogorov-Smirnov confidence level	n.a.	93%
Normal distribution (bought-ins)		
mean μ	0.510	
standard deviation σ	0.120	

Table 5 Results from the calculated (Cohen) and fitted (direct approach) lognormal distributions of the hammer ratios $h = P / FV$ for Ed Ruscha's auction results for drawings, paintings and photographs until 30.11.2012. P is the hammer price and FV the pre-sale geometric mean of upper and lower estimate U and L, respectively. The procedures to find the distribution considers all sold works above a hammer ratio threshold $t_b = 0.857$ (391 data points). The result from the Kolmogorov-Smirnov test for the direct approach is listed together with the confidence level at which the critical value for the rejection of the null-hypothesis is reached. However, the test value should be considered as *indicative* since the parameterisation of the lognormal distribution is based on the empirical distribution and is therefore not independently determined. The Kolmogorov-Smirnov test is not applicable to the functional approach by Cohen. The number of bought-ins (132 data points) is considered according to the assumed lognormal distribution found by Cohen's approach and fitted stand-alone to a normal distribution.

5 Conclusion

In this thesis statistical properties of art pricing models, including price indices, have been examined. Price indices from three different providers and three pricing models have been considered.

5.1 Art Indices

For two of the price indices the model values were compared for Ed Ruscha's *Anchor Stuck in Sand* (1990), which was offered at auction on 7 March 2013 for an estimate between USD 600'000 and USD 800'000 at Phillips in New York. Artprice Indicator® predicted a market value of around USD 200'000, whereas artnet's Report proposed a value of USD 620'000, which was in-line with the auction house's pre-sale estimate. The work was then sold for USD 456'500 without the buyer's premium. While Artprice's proposal is obviously much lower than the auction house's pre-sale estimate, we argued that artnet's market value is questionable due to the application of comparable works for the price determination. In the case of *Anchor Stuck in Sand,* the choice made for the comparable works was not convincing.

The third price index, the AMR Contemporary Art Index, is smoothed through a process of data selection and building averages. The Art Market Research indices make use of the average of single artists' prices at auctions over 12 consecutive months and include the 80% central values only, i.e. excluding the 10% outliers for the lowest and highest prices. De-smoothing the Contemporary art index increased its yearly volatility from 14.6% to 28.8%. Contemporary art prices are rather volatile!

The main take-away from these results is the reminder to be careful when using an art price index without knowing the underlying methodology. Each of the above results can be explained and understood. Knowledge of the methodology allows to judge on the outcome and to decide if and how it should be included in any expectation or decision.

5.2 Art Pricing Models

In this paper Skate's art asset pricing model has been transformed from a simple and heuristic model for the determination of an artworks' fair value into the form of a hedonic regression and found that the price movement over time is associated with the consumer price inflation. Furthermore, the idiosyncratic error term can be interpreted as an irrational premium, which shows the buyer's willingness to over- or underpay the fair value of the artwork. Essentially the model suggests that the reasonable expectation on the price development is a protection against inflation and that irrational behaviour may or may not occur, and is not predictable.

Two other pricing models deal with the expected loss in case the artworks are used as a security for backing bank loans. In the model of Campbell and Wiehenkamp (2009) time-invariant returns from repeat-sales build the base for a price distribution function which is used in the simulation of the loss function. The determination of the distribution function is reiterated with the repeat-sales data from Skate's Art Market Research finding that although the fitted function was well shaped and passed three statistical tests to a high degree, the data is biased and had to be restricted to the returns from artworks at the upper end of the price range. The bias stems from two aspects: Due to the index construction the works at the lower end of the price range show no negative returns. And as usual, the bought-ins at auctions are not considered since their potential transaction price, i.e. the highest bid, is not published.

Since the valuation of securities for backing a bank loan is a delicate task, we strongly recommend to differentiate the artworks according to their price ranges. Scorcu and Zanola (2011) found in a study on Picasso paintings that the so-called masterpieces outperform the other works. In our opinion there is some evidence that this might be the case, but due to statistical uncertainties the conclusion is not as decisive as Scorcu and Zanola present it.

In the model by McAndrew and Thompson (2007) the attempt has been made to estimate the price distribution based on the ratio of the hammer price to the pre-sale auction house estimate, and to include the bought-ins. Their model was applied in this research to the auction results for drawings, paintings and photographs of Ed Ruscha finding that the model fits well with this data sample. Furthermore, a distribution for the bought-ins was determined, which we did not find in this explicit form in the literature. The methodology of McAndrew and Thompson (2007) has the advantage of including all transactions, and considers the bought-ins as important information for the expectation of a potential price decline. As a strong assumption it considers the pre-sale estimate of the auction house (or from any other evaluator) as *true* fair value.

In the application of pricing models the underlying data is as important as the understanding of the methodology if not even more important. In the case of Ed Ruscha, one is in the beneficial situation of dealing with a rather homogeneous portfolio although different media are included. If the selection of works were restricted to paintings, the homogeneity would even increase whereas the statistics would suffer from the lower number of artworks considered. A careful choice is strongly recommended.

5.3 Extensions

Possible extensions and further investigations could cover the following topics:

1) **Model of income inequality, lagged equity return and art price development**
   Goetzmann et al. (2010b) show by using the repeat-sale information from the database established by Renneboog and Spaenjers (2013), which contains transaction data since 1905, that art returns can be explained to a high degree by the income inequality and the one-year lagged equity return. The income inequality is defined as the share of total income received by the top 0.1%-quantile of the global income.
   A possible extension of the model would be the inclusion of real estate market returns as an explanation variable, as has been done for the Japanese market by Hiraki et al. (2005). The authors show the relevance of Japanese land prices for both art and equity markets.
   Another extension concerns the earlier version of the same working paper, Goetzmann et al. (2010a), which contains a simple simulation model for the valuation of an artwork by a private investor that has originally been published in Goetzmann and Spiegel (1995). The art price in the model depends on a) the total global income, b) the investor's share in this total and c) the fraction of his or her income he or she is willing to spend for the artwork. The simulation is run over a sample of investors, which varies over time. Again the authors find that art prices rise with increasing income inequality. The assumptions made in Goetzmann et al. (2010a) serve for illustrative purposes and are kept intentionally simple, e.g. the per capita income growth rates and investor's interest in the artwork are both normally distributed with the standard deviation being equal to the respective mean, and the resale rate of the artworks is set to the maximum in each period.
   With the results found in this thesis it would be interesting to refine the simulation model for the return distribution and to consider the bought-ins. Furthermore it could be checked for other price drivers and e.g. include lagged equity and real estate market returns in the simulation.

2) **Correlation between Equities, Gold and Art**[67]

As this paper has shown, the volatility of art prices can be substantial. Why should an investor consider artworks as part of his asset portfolio? In Campbell (2008) and Deloitte (2011) it is indicated that the correlation between equity markets and art price developments is rather low. However, after the financial crisis in 2007/08 the art market dropped substantially (fig.5), while the gold price had positive returns throughout this period.[68] It would be interesting to consider a portfolio out of equities, gold and art.

3) **Bought-ins as indicator for a future price decline**[69]

Assume that an artwork is brought to auction. The time period until it is auctioned is typically a few months. Assume further, that after the price building is established and the catalogue is in print, the financial market crashes. The seller decides to leave the artwork at auction with the previously agreed lower and upper estimates. In case there is no guarantee, the artwork is probably not sold. If the artwork were brought to auction after the financial crisis, it is very likely that the auction house's estimate would be lower than before the crash. I.e. the bought-in rate can probably serve as an indicator for future price declines at auctions. To support the argument we show in Figure 17 the bought-in rates of Ed Ruscha compared to his price index from Artprice. There is a peak in the price index at the beginning of 2008 followed by a strong price decline in 2008 and 2009, and a bought-in rate peak in 2008, which decreased again in 2009 and 2010. Due to the yearly aggregation no further timing details are readily available. Interestingly, in 2011 a sharp increase of the bought-in rate can be observed, which only partially declined in 2012.

Ashenfelter and Graddy (2011) find a negative correlation of sale rates, i.e. the counterpart of the bought-in rate, with lagged price indices. Their interpretation says that the bought-in rate *follows* the price index, whereas we would rather guess that a bought-in rate increase *precedes* a price decline. It would be interesting to investigate this question.

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[67] Based on discussions with Fabian Bocart, Quantitative Research Director at Tutela Capital.
[68] See e.g. the London Gold Market Fixing prices in US Dollars, Bloomberg ticker: GOLDLNPM Index.
[69] Cf. note 67.

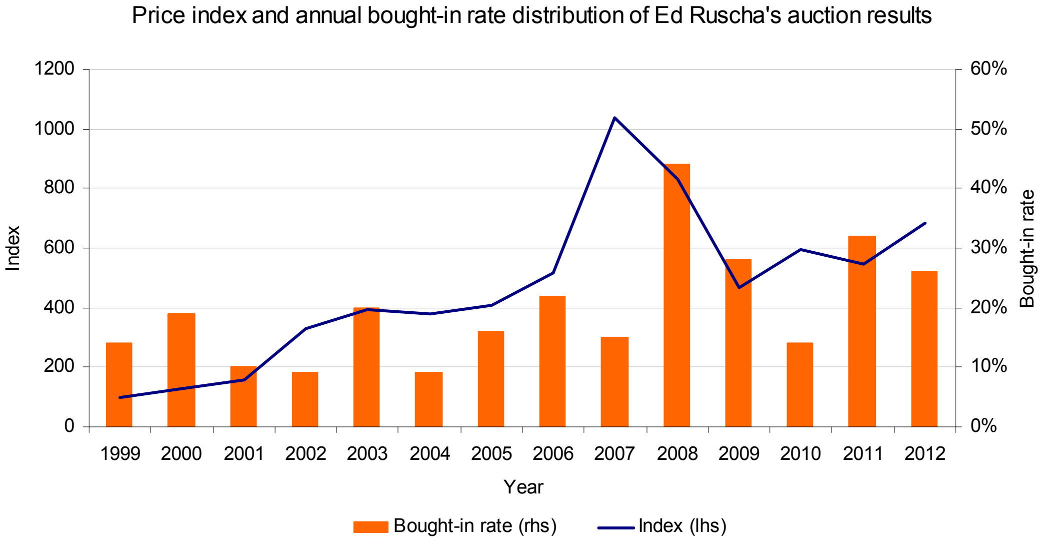

Figure 17 Annual Price Index (dark blue line; left-hand scale) and bought-in rate (orange bars; right-hand scale) for Ed Ruscha's auction results excluding prints.
Data is taken from: www.artprice.com, last access on 10.5.2013.

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Appendix

Appendix A: Comparison of Ed Ruscha's photographs to paintings and drawings at auctions

Photography is an essential medium for conceptual artists like Ed Ruscha in order to realise their ideas. It is, however, unclear whether the photographs are fully recognised in the art market, or whether they are traded with a discount compared to drawings and paintings.

In order to find out the position of photographs compared to unique works we study in this Appendix the prices paid for Ed Ruscha's artwork at auctions. As methodology, the hedonic regression analysis is used and different regressions are performed with distinct features of the artworks as describing variables: Medium, size of the artwork and the number of editions in the case of photographs. In addition, the price development reflected by the index of Ed Ruscha's artworks from Artprice starting in 2000 is referred to, as well as the Art Market Research (AMR) Contemporary Art Index as a proxy before the year 2000.

The results show that with the three variables, including size of the artwork, the number of editions in the case of photographs and the index, the auction prices can be explained to 65%. Further reducing the number of variables and neglecting the photograph's editions reduces the quality of the regression and gives an unsatisfactory fit, especially for the photographs. Therefore, it can be concluded from the regressions that the art market indeed distinguishes the photographs from drawings and paintings insofar as it does take the editions of photographs into account.

Data Selection

Ed Ruscha works in several media where photography is the most important one beside paintings, and his photobooks were important contributions in the early days of conceptual art.[70] For the comparison with photographs, drawings and paintings are taken into account. Ruscha's photographs are closer in unique qualities to his paintings and drawings than to his prints and multiples. The latter are therefore neglected.

The data selection is based on the same original sample as described in Chapter 4.3.2. In total 731 entries were retrieved: 176 drawings, 473 paintings and 82 photographs. 19 entries were eliminated due to quality doubts:

[70] See e.g. Coleman (1972). See also Chapter 4.3.1 for an introduction to Ed Ruscha.

- One entry due to a unrealistic low estimate,
- 18 sold works due to uncertainties whether the prices shown are hammer prices (as indicated) or whether they include the buyer's premium.

Disregarded here are works that have not been sold, i.e. the bought-ins, since the price development is considered by an art index, which makes it necessary to have an auction price. The bought-ins are in total 137 auction records. The bought-in rate differs substantially per category: Whereas it is 16% and 19% for drawings and paintings, respectively, the rate is 30% for photographs. This leads to a restricted number of photographs thus reducing this already limited data sample even further.

Next to consider are only works where information on the size is available, calculated as the area of the artwork, which eliminates 10 works. Dropped from consideration are photographs without the number of editions indicated, which reduces the sample by further 15 photographs. One photograph is excluded due to its extraordinary large edition of 1200 prints and a corresponding low price of USD 500.

Since the considered price development index only started in 1988, five earlier artworks are excluded.

Finally, there are 545 valid auction records which are shown per category in Table 6, together with the ratio of the final to the initial set.

Ed Ruscha's auction results	**Total**	**Drawings**	**Paintings**	**Photographs**
Initial set	**731**	**176**	**473**	**82**
- Quality issues	19	5	14	0
- Bought-ins	137	27	85	25
- No information on size	10	0	5	5
- No information on edition	15	0	0	15
- Large edition	1	0	0	1
- Auctioned before 1988	5	2	3	0
Final set	**545**	**143**	**366**	**36**
Ratio of final to initial set	75%	84%	77%	44%

Table 6 Ed Ruscha's auction results for drawings, paintings and photographs until 30.11.2012. The second column contains the sum of the figures in the columns three to five. The second row shows the initial data set that has been retrieved from Artprice on 26 and 27 December 2012. The rows three to eight contain the numbers of excluded records due to quality issues, missing prices (i.e. bought-ins), missing information on size and (photo) edition, too large edition size and auctions before 1988, respectively. The ninth row (bold) represents the final data set that is used in the analysis, and the last row shows the ratio of the final to the initial set.
Data is taken from: www.artprice.com, last access on 27.12.2012.

Appendix

Figure 18 shows the distribution of the auction prices of the final set per category compared to the prices (x-axis). The prices are mapped according to a logarithm with base 2, i.e. $\log_2(Price)$, in order to better visualise the lower priced artworks, which dominate the sample. It is obvious that the paintings (dark blue bars; bottom layer) cover the higher price region, whereas the drawings (light green bars; middle layer) are most prominent between USD 20'000 and USD 200'000, and the photographs (orange bar; top layer) are limited to below USD 200'000.[71]

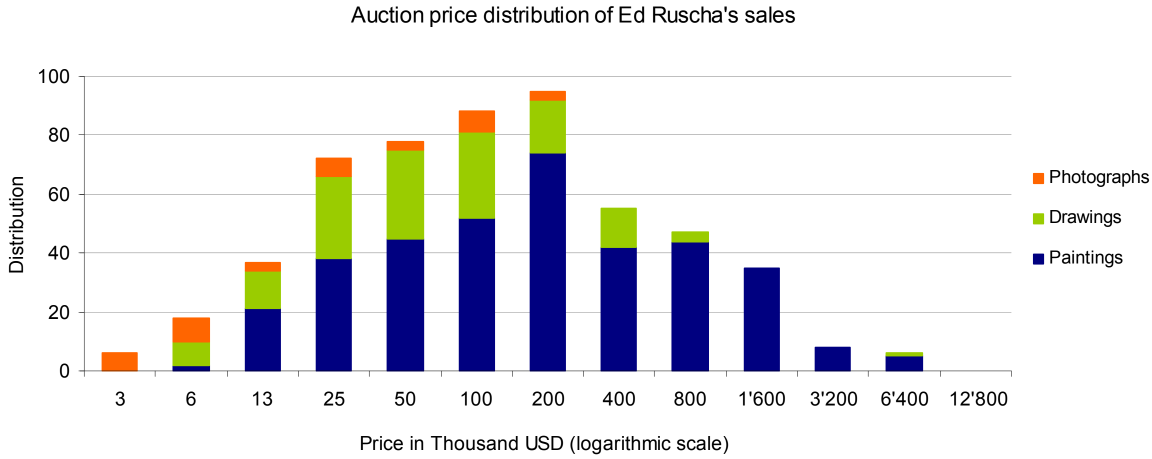

Figure 18 Distribution of auction prices for artworks of Ed Ruscha. The prices (x-axis) are scaled according to a logarithm with base 2, i.e. $\log_2(Price)$. The paintings are the dark blue bars (bottom layer), the drawings are the light green bars (middle layer) and the photographs are the orange bars (top layer). The chart shows the final set in Table 6 (545 data points).
Data is taken from: www.artprice.com, last access on 27.12.2012.

Hedonic Regression

As described in Chapter 2, hedonic regression is a technique used to find relations between features of a good and a variable to be explained. Typically this variable is the (transaction) price, and the attributes of the good are taken as measureable and fixed input in order to establish a predictable relation.

The goal of this Appendix is to explain the prices at auction of Ed Ruscha by means of a few features of drawings, paintings and photographs. The choice of the variables is:

- **Medium**: Paintings are worth more than drawings, and both are expected to be higher priced than photographs.
- **Size**: Intuitively one expects a higher price for a larger artwork.

[71] The reader should keep in mind that the prices are *nominal* prices not adjusted for inflation or for a price development of the work between the date when the auction took place and today.

- **Number of editions in the case of photographs**: The larger the edition the lower the price for a single print.
- **Index**: Market development of Ed Ruscha's artworks to adjust for the historical prices in order to get the present value of the auctioned works.

The low number of variables should allow for a straightforward explanation of the results. Increasing the number of variables most likely gives a higher degree of numerical precision. However, the interpretation gets harder when more variables are considered.[72]

The mathematical expression for the hedonic regression of the price of an artwork reads[73]

$$\begin{aligned}\ln(Price) &= \beta_0 + \beta_1 \cdot \mathbf{1}_{Drawing} + \beta_2 \cdot \mathbf{1}_{Painting} + \beta_3 \cdot \ln(Size) \\ &\quad + \beta_4 \cdot \ln(Photo\ edition) + \beta_5 \cdot \ln(Index) + \varepsilon\end{aligned} \tag{A.1}$$

where $\ln(X)$ is the natural logarithmic function and $\mathbf{1}_Y$ is the indicator function for drawings and paintings, respectively. E.g.

$$\mathbf{1}_{Drawing} = \begin{cases} 1 & \text{in case medium} = \text{drawing} \\ 0 & \text{otherwise} \end{cases}.$$

$Size$ is the area of the artwork, i.e. height times width, $Photo\ edition$ is the number of prints in the case of photographs, and $Index$ reflects the price development of Ed Ruscha's artwork explained below. β_i are the parameters to be determined, valid for all the art works in the data set, and ε is a residual term, which contains the unexplained part of the price.

After applying the exponential function $\exp(x)$ to both sides of eq.(A.1) we obtain the following equation for the price relation,

$$Price = \exp\left(\beta_0 + \beta_1 \cdot \mathbf{1}_{Drawing} + \beta_2 \cdot \mathbf{1}_{Painting} + \varepsilon\right) \cdot Size^{\beta_3} \cdot \left(\text{Photo edition}\right)^{\beta_4} \cdot Index^{\beta_5} \tag{A.2}$$

The influence of the parameters β_i on the price can be read off from eq.(A.2).

The price development is represented by the index of Ed Ruscha's artworks from Artprice starting in 2000. For the pre-2000 time period we use as a proxy the Art Market Research (AMR) Contemporary Art Index starting in 1987.[74] Since the index is built from price averages, which lead to a smoothed time-series, we use a de-smoothed version of the index, which has a substantially higher volatility than the reported time-series.[75] Ruscha's Artprice index is regressed to the de-smoothed AMR

[72] Potential extensions include the subject of a work or the technique used in paintings.
[73] Ginsburgh et al (2006).
[74] See Chapter 3.3.
[75] Ibid.

Appendix

Contemporary Art Index for the period in which both exist in parallel, from 2000 to 2010. The resulting excess return α and the market factor β_{Market} are assumed to be valid also before year 2000,

$$Index_{Ed\ Ruscha} = \alpha + \beta_{Market} \cdot Index_{Contemporary\ Art} + \varsigma ,$$

with the idiosyncratic error term ς. At the 1%-confidence level the parameters are

$\alpha = 0$ and $\beta_{Market} = 0.700$ (with an adjusted $R^2 = 0.41$).

The goodness-of-fit, expressed in R^2, is not very high, essentially due to 2008: In that year the Contemporary Art Index still increased, while the prices for Ruscha's artwork already started decreasing.

The final index used in the analysis is constructed in the following way

$$Index = \begin{cases} 0.7 \cdot Index_{Contemporary\ Art} & \text{before year 2000} \\ Index_{Ed\ Ruscha} & \text{year 2000 and later} \end{cases} .$$

Results

The information on the final data sample is put into eq.(A.1) and the linear regression is solved. The goal is to find the parameter set β_i that gives the best description of the auction prices. The procedure used is called Ordinary Least Squares (OLS), which measures the difference, called error, between each individual auction price and the corresponding theoretical price from the model, squares these differences and sums them up. The optimal solution is determined by the set of parameters β_i that minimises the sum of the squared errors. Once a solution is found a check is performed on the relevance of the describing variables and as many irrelevant ones as possible are eliminated. An optimisation for the parameters β_i has to be conducted on each restricted set of parameters.

For the goodness-of-fit we refer to the *adjusted* R^2, which is between zero and one, and the higher the value the better the precision. $R^2 = 1$ means perfect coincidence. For the rejection of variables we use the p-value, which indicates at which level the variable is significant, i.e. should *not* be rejected. The lower the better: Significance is usually associated with a p-value below 5%. As a tool for solving the linear regression we use Microsoft Excel.[76]

The first regression is done with all the parameters β_i and shown in Table 7. For the intercept and for the distinction between the media drawings and paintings it can

[76] For a tutorial explanation of the regression tool in Microsoft Excel and of the quantities mentioned in this Chapter we refer to Orlov (1996).

be seen that the coefficients are not significantly different from zero since the standard errors are larger than the coefficients themselves. This is confirmed by the probability to reject these coefficients. On the other hand the size, the amount of editions for photographs and the price development are highly significant, all at or below the 1%-level and with very small errors, except the photo edition variable. The quality of the fit is good, $R^2 = 65.0\%$, i.e. on average almost two thirds of an auction price can be explained by the chosen parameters.

Variable	Parameter	Coefficient	Standard error	Significance
Intercept	β_0	0.803	1.237	51.6%
Drawing	β_1	-1.050	1.150	36.1%
Painting	β_2	-0.971	1.148	39.8%
Size	β_3	0.582	0.036	< 1%
Photo edition	β_4	-0.848	0.335	1.2%
Index	β_5	1.167	0.050	< 1%
	R^2	0.650		

Table 7 Result from the first regression with all parameters in eq.(A.1). For each explaining variable we indicate the coefficient, the standard error and the level of significance. The last row contains the quality of the fit.

Next, the first three parameters β_0 to β_2 are set to zero and the regression is repeated with the remaining parameters. The result is shown in Table 8. While the coefficients for the size and the index remained almost unchanged with even smaller errors than in the first regression, the relevance of the photograph’s editions increased substantially now having an error comparable to the other two variables. Since the quality of the fit, $R^2 = 65.2\%$, remained almost unchanged compared to the first regression, the second regression with less parameters is superior to the first regression. All variables are highly significant. The resulting logarithmic prices of this regression are shown in Figure 19 compared to the corresponding auction prices. The paintings are on the upper end of the scale, and the photographs at the lower end.

As a third and last regression the question is whether the number of editions for photographs really matters. Are the photographs treated differently from paintings and drawings? In this case we set $\beta_4 = 0$, and repeat the regression with the two remaining variables.

Appendix

Variable	Parameter	Coefficient	Standard error	Significance
Intercept	β_0	0	-	-
Drawing	β_1	0	-	-
Painting	β_2	0	-	-
Size	β_3	0.572	0.022	< 1%
Photo edition	β_4	-0.563	0.048	< 1%
Index	β_5	1.150	0.034	< 1%
	R^2	0.652		

Table 8 Result from the second regression with the parameters β_0 to β_2 set to zero in eq.(A.1). For each of the remaining explaining variables we indicate the coefficient, the standard error and the level of significance. The last row contains the quality of the fit.

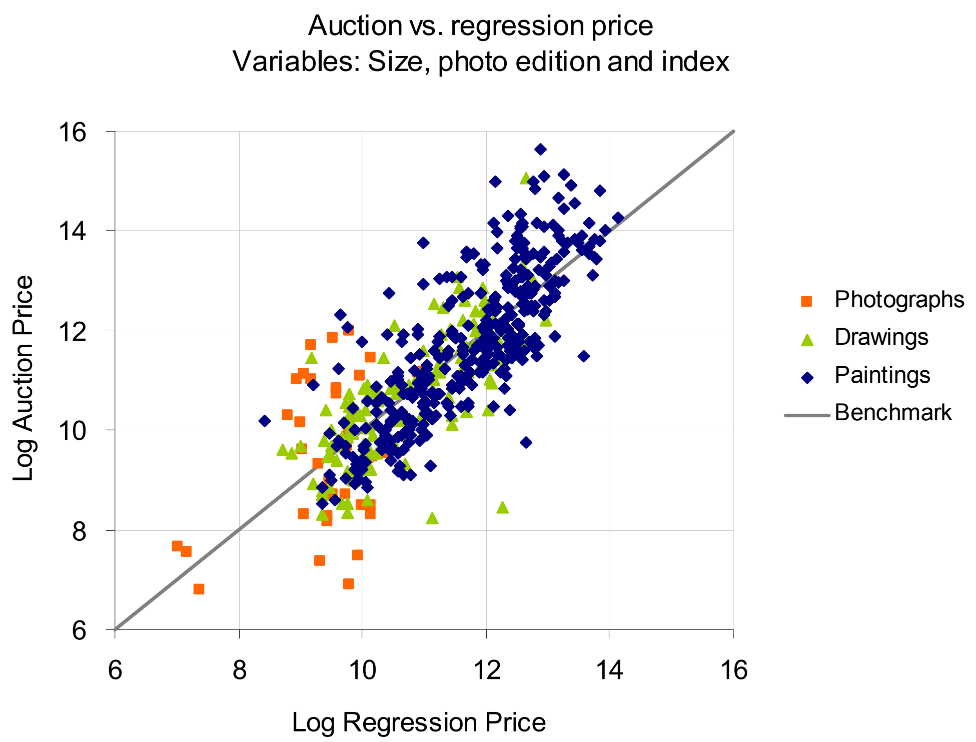

Figure 19 Distribution of the natural logarithm of auction prices for artworks of Ed Ruscha (y-axis) compared to the prices determined in the second regression analysis (x-axis), see Table 8. The paintings are the dark blue diamonds, the drawings are the light green triangles and the photographs are the orange squares. The benchmark line indicates the perfect coincidence between regression and auction price.
Auction data is from: www.artprice.com, last access on 27.12.2012.

The result is shown in Table 9. Again the coefficients for the size and the index remained stable, as well as their errors, but for this regression the goodness-of-fit decreased to $R^2 = 56\%$. And as can be seen in Figure 20 this is entirely due to the worse fit for the photographs which strongly deviate from the indicated benchmark as perfect coincidence between the observed auction prices and the calculated prices from the regression. Therefore the third regression is considered less reliable than the second regression.

Variable	Parameter	Coefficient	Standard error	Significance
Intercept	β_0	0	-	-
Drawing	β_1	0	-	-
Painting	β_2	0	-	-
Size	β_3	0.566	0.021	< 1%
Photo edition	β_4	0	-	-
Index	β_5	1.159	0.032	< 1%
	R^2	0.563		

Table 9 Result from the third regression with the parameters β_0 to β_2 and β_4 set to zero in eq.(A.1). For each of the remaining explaining variables we indicate the coefficient, the standard error and the level of significance. The last row contains the quality of the fit.

Figure 20 Distribution of the natural logarithm of auction prices for artworks of Ed Ruscha (y-axis) compared to the prices determined in the third regression analysis (x-axis), see Table 9. The paintings are the dark blue diamonds, the drawings are the light green triangles and the photographs are the orange squares. The benchmark line indicates the perfect coincidence between regression and auction price.
Auction data is from: www.artprice.com, last access on 27.12.2012.

Appendix

Interpretation

Consequently, the interpretation of the results is done on the second regression as listed in Table 8. The coefficients indicate that the size and the index have both a positive impact on the price, while the number of editions of photographs influences the price downwards. The coefficient for the index is very close to one, which comes as no surprise since it is assumed that the index, at least after the year 2000, reflects the price movement of Ed Ruscha's artworks. Also the other two coefficients are explainable by intuition: The larger the size of the artwork, the higher the price, and the larger the photograph's edition, the lower the price.

The results are transformed into eq.(A.2) for the numerical interpretation,[77]

$$Price = Size^{0.57} \cdot (Photo\ edition)^{-0.56} \cdot Index^{1.15} .$$

In a rough approximation, the price formula can be reduced to square roots of the size and the edition and a slightly increased dependence on the index,[78]

$$Price \approx \frac{\sqrt{Size}}{\sqrt{Photo\ edition}} \cdot Index^{\frac{5}{4}} .$$

which means that the price increases with the square root of the area, and decreases with the square root of the number of photograph editions. The former finding is in line with the practice to sum height and width of an artwork in order to determine the price of a work, rather to scale it with the area. The latter tells us that although the market is aware of the number of prints it does not linearly account for it. For example: If an edition is *four times* as numerous as another one, the price for a single print out of this "big" edition is expected to be *half the price* as for a photograph out of the edition with fewer prints.

An extension of this study could include the subject of a work as a regression parameter. Or to include a more detailed differentiation for the largest category, the paintings, by e.g. including the paints (oil, acrylic, etc.) as explaining variables in order to further improve the goodness-of-fit. A further possibility is the determination of the regression parameters per medium, and to check whether they are medium-dependent.

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[77] The residual term ε is dismissed for this discussion.

[78] If the coefficients of the size and the edition are set to $\beta_3 = 0.5$ and $\beta_4 = -0.5$, the regression results in $\beta_5 = 1.25 \pm 0.01$ for the index coefficient.

Appendix B: Remark on logarithmic properties

Chapter 2 of this thesis assumes that price indices P_{it} can be expressed by means of the natural logarithm. Other functional forms of P_{it} are possible, however, the logarithm has several properties that make it the preferred function:

- The ranking is conserved, i.e. if $P_{it} < P_{jt}$ this holds also true for the logarithm, $P_{it} < P_{jt} \Leftrightarrow \ln(P_{it}) < \ln(P_{jt})$,
- Multiplicative terms can be expressed as sums, i.e. $\ln(a \cdot b) = \ln(a) + \ln(b)$,
- There is a simple first-order Taylor-approximation for small values, i.e. $\ln(1+c) \approx c$ for $c << 1$.

Since returns are geometric functions, the second property is highly welcome. Assume a compounded return r for the period from t to T. The price development may be formulated as $P_T = P_t \cdot \exp[r \cdot (T-t)]$, which is transferred into

$$\ln(P_T) = p_T = p_t + \ln\{\exp[r \cdot (T-t)]\} = p_t + (T-t) \cdot r .$$

Appendix C: Rating system of the Artprice Indicator®

In Chapter 3.1 we consider the Artprice Indicator®. The goal of the indicator is to give a present value for a work which has been auctioned. The indicative price comes with a rating system for the quality of the estimate, called degree of relevance. This quality statement takes into account five numerical factors, which are all mapped to be between zero and one. The five factors are:[79]

- C_1: The multiple correlation coefficient R^2 that explains the degree of price variation by the price regression. The closer to one, the better the model.
- C_2: A ranking based on the degrees of freedom dof left after fitting the data
  $$C_2 = \begin{cases} 0.5 & if\ dof < 100 \\ 1 & if\ dof \geq 100 . \end{cases} .$$
- C_3: A ranking based on the index type used in the price regression
  $$C_3 = \begin{cases} 1 & for\ artwork\ price\ index \\ 0.8 & for\ medium\ price\ index \\ 0.6 & for\ artist\ price\ index \end{cases} .$$

[79] Artprice Indicator Methodology, web.artprice.com/indicator/howto?l=en, last access on 20.2.2013.

- C_4: The significance of the yearly price index parameter determined on the probability of rejection of the hypothesis that the parameter could be neglected in the price regression. The exact mapping is not indicated.
- C_5: The volatility of the price index reflected by the first-order autocorrelation. This methodology assumes that the larger the autocorrelation the smaller the volatility, and the less erroneous the price index.

The degree of relevance is the geometric mean of the C_i,

$$C = (C_1 \cdot C_2 \cdots C_5)^{1/5}$$

The ranking is given by a system between three and five stars. Results with less than three stars are not published,

$$C = \begin{cases} \bigstar\bigstar\bigstar\bigstar\bigstar & 0.8 \le C \le 1 \\ \bigstar\bigstar\bigstar\bigstar\text{☆} & 0.6 \le C < 0.8 \\ \bigstar\bigstar\bigstar\text{☆☆} & 0.4 \le C < 0.6 \end{cases}.$$

C_1, C_2 and C_3 are described and derived in an unambiguous way, and even with no further insights a rough guess of the range of the parameters seems possible. For C_4 the mapping is not obvious. If it is the $p-$value directly, then a high rejection probability may decrease the final parameter C substantially. In the case of C_5, a higher autocorrelation yields indeed a lower volatility, but that does not mean that the price development is in fact less error prone.[80]

Appendix D: Removal of autocorrelation in time series

Following Okunev and White (2003), the reported returns are adjusted to remove the autocorrelations of the AMR Contemporary art index. The following form of a return time series r_t is assumed:

$$r_{0t} = (1-\alpha) \cdot r_{mt} + \sum_{i=1}^{N} \beta_i \cdot r_{0(t-i)},$$

where r_{0t} is the reported return at time t, and r_{mt} is the underlying (unreported) return at time t. m is the number of adjustments made to r_{0t} and N is the number of prior returns that have been corrected in r_{mt}. The sum over the β_i's is constrained to

[80] See also the discussion of the implication of higher autocorrelation in Chapter 3.3 and Appendix D.

Appendix

$\alpha = \sum_{i=1}^{N} \beta_i$.[81]

The methodology of the elimination of the first order autocorrelation can be traced back to Geltner (1991) who used the procedure in order to de-smooth real estate returns. The relation to get the first corrected return series r_{1t} reads:

$$r_{1t} = \frac{r_{0t} - c_1 \cdot r_{0(t-1)}}{1 - c_1},$$

where c_1 is the coefficient to be determined. The derivation of the general solution for c_1 can be found in Okunev and White (2003). In order to remove the first order autocorrelation completely, c_1 becomes

$$c_1 = \frac{(1 + a_{02}) \pm \sqrt{(1 + a_{02})^2 - 4 \cdot a_{01}^2}}{2 \cdot a_{01}}, \tag{D.1}$$

where a_{01} and a_{02} are the first and second order autocorrelations of the reported return series r_{0t} , respectively,

$$a_{0q} = Corr[\,a_{0t}, a_{0(t-q)}], \quad q = 1, 2\,.$$

For later use a_{mq} is defined as the q[th] order autocorrelation of the m times corrected return series r_{mt} ,

$$a_{mq} = \mathrm{Corr}[a_{mt}, a_{m(t-q)}]\,.$$

Eq.(D.1) only has a real solution if $4 \cdot a_{01}^2 \leq (1 + a_{02})^2$. Assuming for a moment that the higher autocorrelations are negligible, r_{0t} can be written as

$$r_{0t} = (1 - c_1) \cdot r_{1t} + c_1 \cdot r_{0(t-1)},$$

and the corresponding variance is

$$(1 - c_1)^2 \cdot \mathrm{Var}[r_{1t}] = \mathrm{Var}[r_{0t} - c_1 \cdot r_{0(t-1)}] \;\Rightarrow\; \mathrm{Var}[r_{1t}] = \frac{1 + c_1^2 - 2 \cdot c_1 \cdot a_{01}}{(1 - c_1)^2} \cdot \mathrm{Var}[r_{0t}]\,.$$

Since $-1 \leq a_{01} \leq 1$ we have for $c_1 \geq 0$:

$$1 + c_1^2 - 2 \cdot c_1 \cdot a_{01} \geq (1 - c_1)^2,$$

[81] Note that in Okunev and White (2003) the sum over the β_i's is erroneously given to be equal to $(1 - \alpha)$.

and $\mathrm{Var}[r_{1t}] \geq \mathrm{Var}[r_{0t}]$, i.e. the variance of the corrected return series is larger than the reported return series in case c_1 is positive. Therefore, having a positive first order autocorrelation underestimates the volatility in the smoothed (reported) returns.

We can repeat the removal of the autocorrelation to the second order, now working on the corrected series r_{1t}. The two times corrected return series r_{2t} becomes analogously,

$$r_{2t} = \frac{r_{1t} - c_2 \cdot r_{1(t-2)}}{1 - c_2}$$

and c_2 is

$$c_2 = \frac{\left(1 + a_{14}\right) \pm \sqrt{\left(1 + a_{14}\right)^2 - 4 \cdot a_{12}^2}}{2 \cdot a_{12}}$$

with a real solution if $4 \cdot a_{12}^2 \leq \left(1 + a_{14}\right)^2$. An approximate form of the de-smoothed returns can be derived by neglecting cross-terms of second order,

$$r_{0t} = \left(1 - c_1 - c_2\right) \cdot r_{2t} + c_1 \cdot r_{0(t-1)} + c_2 \cdot r_{0(t-2)} + O\left(c_1 \cdot c_2\right).$$

Again, the variance $\mathrm{Var}[r_{2t}]$ is larger than $\mathrm{Var}[r_{1t}]$ in case c_2 is positive.

Due to the removal of the second order autocorrelation and the corresponding change in the time series, it cannot be avoided that again a first order autocorrelation is introduced into the return series r_{2t}. Okunev and White (2003) describe in detail the iterative procedure to remove again the first order autocorrelation in the same manner as shown above. Numerically, however, the effects are rather small. In AMR's Contemporary art index the autocorrelations are removed straightforwardly finding that the newly induced autocorrelations can be left uncorrected since the new autocorrelation coefficients are below the 95% confidence level for a normal distribution with zero mean and a standard deviation of $\frac{1}{\sqrt{T}}$, where $T = 305$ is the total number months considered.[82] The critical band at 95% confidence level lies therefore within $\pm \frac{1.96}{\sqrt{305}} = \pm 0.112$.

In a generalised form we can write the returns r_{mt} and the corresponding coefficient c_{mq}, which eliminates the q$^{\text{th}}$ order autocorrelation in the m times corrected return series r_{mt},

[82] See Diebold (2004), p. 137–140.

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$$r_{mt} = \frac{r_{(m-1)t} - c_{mq} \cdot r_{(m-1)(t-q)}}{1 - c_{mq}}$$

and

$$c_{mq} = \frac{\left(1 + a_{(m-1)2\cdot q}\right) \pm \sqrt{\left(1 + a_{(m-1)2\cdot q}\right)^2 - 4 \cdot a_{(m-1)q}^2}}{2 \cdot a_{(m-1)q}}$$

with a real solution if $4 \cdot a_{(m-1)q}^2 \leq \left(1 + a_{(m-1)2\cdot q}\right)^2$. For each $c_{mq} > 0$ the variance increases and for each $c_{mq} < 0$ the variance decreases.

Appendix E.1: Expected loss for art backed loans

In Campbell and Wiehenkamp (2009), the expected loss EL for the bank due to the loan is written as

$$EL = \int_O^T q(t) \cdot \nu(t) \cdot dt ,$$

where $q(t)$ is the time-dependent default probability function for the specific borrower, and $\nu(t)$ is the loss function,

$$\nu(t) = \begin{cases} P_0^{Loan} \cdot \exp(-r_f \cdot t) \cdot \left[1 - \tilde{\kappa}_t \cdot \min\left(\frac{P_t^{Art}}{P_0^{Loan}}, 1\right) + i_t\right] & in\ case\ of\ default\ at\ time\ t \\ 0 & otherwise \end{cases}$$

P_0^{Loan} is the loan valued at time $t = 0$, $\exp(-r_f \cdot t)$ is the discount factor for the present value of the loan, and i_t is the accrued interest. $\tilde{\kappa}_t$ reflects the availability of buyers in case of an urgent liquidation of the artwork, and p_t^{Art} is the assumed price of the underlying artwork at time t.

Campbell and Wiehenkamp (2009) consider data from Sotheby's London with mostly Impressionist paintings, Victorian pictures, Old Masters, 16th century British paintings and Modern Art. The final data set contains 398 repeat-sales pair and its empirical return distribution is fitted to the normal, logistic and t-Student distributions. The authors find that out of the three statistical functions the t-Student distribution gives the closest description of the data. This specific data fitted t-Student distribution is used to simulate the loss function $\nu(t)$.[83]

[83] The buyer availability function $\tilde{\kappa}_t$ is assumed to be a Poisson distribution, see Campbell and Wiehenkamp (2009).

Appendix E.2: Artworks as collateral values in bank loans

The approach of McAndrew and Thompson (2007) considers the loss given default in case the borrower is unable to pay back the bank loan (or pay the interest). The authors assume that the valuation of the artwork can be done based on the auction history of a single artist, the period for different artists or other criteria leading to comparable art objects. The method is time-invariant, i.e. it assumes that the observed prices in relation to the pre-sale estimates are unconditional to the point in time. The loss given default LGD is defined as

$$LGD = 1 - \frac{1}{LTV \cdot \int_0^{LTV} g(x) \cdot dx} \cdot \int_0^{LTV} x \cdot g(x) \cdot dx \,,$$

where LTV is the loan-to-value ratio and $g(x)$ is the distribution function of the auction prices.

Appendix F: Statistical tests

Three tests are performed for the cumulative distribution function in Chapter 4.2 in order to test the null-hypothesis $H_0 : F(x_i) = Y(x_i)$, i.e. that the assumed t-Student distribution $F(x_i)$ is the correct distribution to describe the empiric cumulative distribution function $Y(x_i)$. $Y(x_i)$ is determined from the observations x_i, which are indexed in ascending order, $x_1 \le x_2 \le \ldots \le x_n$, and can therefore be written as

$$Y(x_i) = \frac{i}{n}, \quad i = 1,2\ldots,n \,.$$

Kolmogorov-Smirnov test

In the Kolmogorov-Smirnov test the maximum $d_{\max}$ is determined from the point wise distances between $F(x_i)$ and $Y(x_i)$, and its next neighbour $Y(x_{i-1})$, i.e.

$$d_{\max} = \max(a_1, \ldots, a_n; b_1, \ldots, b_n)$$

with

$$a_i = \left| F(x_i) - \frac{i}{n} \right|, \text{ and}$$

$$b_i = \left| F(x_i) - \frac{i-1}{n} \right| \,.$$

For $n > 40$ a critical distance d_{crit} is given by a simple formula dependent on the confidence level α,

$$d_{crit}(\alpha) = \sqrt{\frac{1}{2 \cdot n} \cdot \ln\left(\frac{2}{\alpha}\right)}.$$

If $d_{\max} > d_{crit}(\alpha)$ then the null-hypothesis H_0 that $F(x_i)$ is the correct distribution for the empirical cumulative distribution function $Y(x_i)$, $H_0 : F(x_i) = Y(x_i)$ has to be rejected at the confidence level α.

Cramér-von Mises and Anderson-Darling tests

Other tests measure the weighted piecewise difference of the area between $F(x)$ and $Y(x_i)$. A convenient way to write the sum of squared differences is

$$Q_n^2 = n \cdot \int_0^1 [F(x) - Y(x)]^2 \cdot \Psi(x) \cdot dF(x),$$

where the weight function $\Psi(x)$ will be determined below, and $Y(x)$ is a step function for the purpose of the integral representation; a transformation of the integral to a sum is then performed for the aim of the calculation.

For the *Cramér-von Mises test* the weight is $\Psi_{CvM}(x) = 1$, and the test measures mainly the deviation around the mean of $F(x)$. The integral can be expressed as a sum,[84]

$$W_n^2 = n \cdot \int_0^1 [F(x) - Y(x)]^2 \cdot dF(x) = \frac{1}{12 \cdot n} + \sum_{i=1}^{n} \left[\frac{2 \cdot i - 1}{2 \cdot n} - F(x_i)\right]^2.$$

The critical values for W_n^2 are tabulated for specific number of observations and confidence levels α.[85]

The *Anderson-Darling test*, finally, is similar to the Cramér-von Mises test; however, it introduces a different weight in the integral,

$$\Psi_{AD}(x) = \frac{1}{F(x) \cdot [1 - F(x)]},$$

which gives more weight to the tails and leads to the squared standardised difference between the empirical cumulative distribution $Y(x)$ and the assumed cumulative distribution function $F(x)$.

[84] See e.g. http://www.aiaccess.net/English/Glossaries/GlosMod/e_gm_goodness_of_fit.htm, last access on 20.1.2013.
[85] See Stephens (1970); Csörgö and Faraway (1996).

Again, the integral can be expressed as a sum,[86]

$$A_n^2 = n \cdot \int_0^1 \frac{[F(x)-Y(x)]^2}{F(x)\cdot[1-F(x)]} \cdot dF(x)$$

$$= -n + \frac{1}{n}\sum_{i=1}^{n}(2\cdot i-1)\cdot\{\ln[F(x_i)] + \ln[1-F(x_{n+1-i})]\}$$

A_n^2 gives a p-value with which the null-hypothesis $H_0 : F(x) = Y(x_i)$ can be directly tested.

Appendix G: Estimate of mean and standard deviation for a truncated normal distribution

The approach by Cohen (1959) considers a normal distribution that is truncated or censored in one of the tails and yields the mean and standard deviation of the complete distribution by means of a functional form. The needed input consists of the sample data above or below the threshold where the distribution is truncated, and of the number of the dismissed data points. It links the mean $\bar{x}$ and the variance s^2 of the upper or lower tail to the mean μ and the variance σ^2 of the full distribution, under the assumption that it is normal distributed. The connection reads

$$\mu = \bar{x} - \lambda(n_l,\zeta)\cdot(\bar{x}-\tilde{t}),$$

$$\sigma^2 = s^2 + \lambda(n_l,\zeta)\cdot(\bar{x}-\tilde{t})^2, \qquad \text{(G.1)}$$

$$\frac{1-\Gamma(n_l,\zeta)\cdot[\Gamma(n_l,\zeta)-\zeta]}{[\Gamma(n_l,\zeta)-\zeta]^2} = \frac{s^2}{(\bar{x}-t)^2},$$

and

$$\lambda(n_l,\zeta) = \frac{\Gamma(n_l,\zeta)}{\Gamma(n_l,\zeta)-\zeta},$$

$$\Gamma(n_l,\zeta) = \frac{n_l}{1-n_l}\cdot\frac{\varphi_N(\zeta)}{\Psi_N(\zeta)},$$

$$n_l = \frac{N_l}{N},$$

where $\tilde{t} = \ln(t_b)$ is the truncation point for the normal distribution function, N_l is the number of truncated data points and N is the complete number of the empirical data

[86] cf. note 84.

points.[87] $\varphi_N(\zeta)$ and $\Psi_N(\zeta)$ are the standard normal density and cumulative function, respectively, evaluated at ζ. Solving the third equation of eq.(G.1) for ζ, and therefore $\Gamma(n_l,\zeta)$, we find $\lambda(n_l,\zeta)$ and can consequently solve the first two equations for μ and σ^2, which then are the location and scale parameters for the lognormal distribution, respectively. In this study we consider the upper tail and have $\bar{x} = 0.279$, $s^2 = 0.123$, $\tilde{t} = -0.154$, $N_l = 299$ and $N = 690$.

[87] Note that in McAndrew and Thompson (2007), p. 598, N_l is incorrectly associated with the number of data points *above* the threshold $\tilde{t}$.